Internal Assessment
M.A./ M.Sc. Semester-III Examination, 2018 (DDE)
Subject: Mathematics (Pure Stream)

Use separate answer-sheet for each paper (**Answer of each paper should be limited to one A4 size page**)

Notation and symbols have their usual meanings

Time: 2 Hours　　　　　　　　　　　　　　　　　　　　　　　　Full Marks: 25

Paper: MPG 301
UNIT-I (Modern Algebra-II)

Answer any **One** question. Only **first answer** will be evaluated.　　　　1×3=3

1. Let H be a normal subgroup of a group G such that $|H| = 2$. Prove that $H \subseteq Z(G)$.　　(3)
2. Show that in a commutative ring with 1, every maximal ideal is prime.　　(3)

Unit-II (General Topology)

Answer any **one** question. Only **first answer** will be evaluated.　　　　1×2=2

1. Let (X, τ) be the product space of a family of topological space $\{(X_\alpha, \tau_\alpha) : \alpha \in \Lambda\}$, Λ being index set. Is each projection map $p_\alpha : X \to X_\alpha$, $\alpha \in \Lambda$, continuous? Justify your answer.　　(2)
2. Is every locally compact space compact? Support your answer.　　(2)

Paper: MPG 302
Unit-I (Graph Theory)

Answer any **one** question. Only **first answer** will be evaluated.　　　　1×4=4

1. (a) Does there exist any simple planar graph with 5 vertices and 11 edges? Justify your answer.
   (b) Give an example of Eulerian graph which is not Hamiltonian.　　(2+2)
2. (a) State Havel-Hakimi theorem.
   (b) Draw the graph of which $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is the adjacency matrix.　　(2+2)

Unit-II (Set Theory-I)

Answer **any one** question. Only **first answer** will be evaluated.　　　　1x1=1

1. Define Cartesian product of a non-void family $\{A_i : i \in I\}$ of non-void sets.
2. State Axiom of choice.

Paper: MPG 303
Unit-I (Set Theory-II and Mathematical Logic)

Answer **any one** question. Only **first answer** will be evaluated.　　　　1x3=3

1. Let α and β be any two ordinal numbers. Does the result $\alpha + \beta = \beta + \alpha$ hold? Support your answer.
2. Determine whether $((A \Leftrightarrow ((-B) \vee C)) \Rightarrow ((-A) \Rightarrow B))$ is a tautology.

Unit-II (Functional Analysis-II)

Answer **any one** question. Only **first answer** will be evaluated. 1x2=2

1. What do you mean by completion of a metric space? Examine if the subspace $\wp[0,1]$ of all real valued polynomials over $[0,1]$ is a closed subspace of $c[0,1]$ with respect to sup norm. (1+1)
2. In $\mathbb{R}^2$ define the following two norms by $\|x\|_1 = \max\{|x_1|, |x_2|\}$ and $\|x\|_2 = |x_1| + |x_2|$, where $x = (x_1, x_2) \in \mathbb{R}^2$. Are the two norms equivalent? Justify your answer. (2)

Paper: MPS 304 (Advanced Functional Analysis-I)
Special Paper –I

Answer **any one** question. Only **first answer** will be evaluated. 1x5=5

1. (a)If p is a semi-norm on a vector space X and if $K = \{x \in X : p(x) < 1\}$, show that K is a balanced and absorbing set in X.
2. For any element x of a topological vector space X, prove that $cl\{x\} = x + cl\{0\}$. (2+3)

   (a) Is $L_1[0,1]$ strictly convex? Justify your answer.

   (b) Let K be a convex absorbing set containing the zero vector of a topological vector space X and let P_k be the Minkowski functional for K over X. If $P_k(x) < 1$, prove that $x \in K$.

(2+3)

Paper: MPS 304 (Differential Geometry of Manifolds-I)
Special Paper –I

Answer **any one** question. Only **first answer** will be evaluated. 1x5=5

1. (a) Define topological manifold.

   (b) Is the tensor product commutative? Support your answer. (2+3)
2. (a) When is a curve on a manifold said to be differentiable?

   (b) Evaluate: $(5dx + 3dy + 4dz) \wedge (dx - 2dy - 3dz)$. (2+3)

Paper: MPS 305 (Operator Theory and Applications-I)
Special Paper–II

Answer **any one** question. Only **first answer** will be evaluated. 1x5=5

1. Let X be a complex Hilbert space and $A, B \in B(X, X)$. If A and B are normal and $AB^* = B^*A$ then show that $A + B$ is normal. If $T \in B(X, X)$ is self-adjoint, show that $\langle Tx, x\rangle$ is real for all $x \in X$. (3+2)
2. (a)Let E_1 and E_2 be two orthogonal projections on the closed subspaces M_1 and M_2 respectively of a Hilbert space X. If $E_1 \le E_2$ then show $\|E_1 x\| \le \|E_2 x\|$, for all $x \in X$.

   (b) Show that annihilator of subset A of the normed linear space X is closed. (3+2)