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M.A./M.Sc. Semester II Examination, 2020 (CBCS)
Subject: Mathematics
Course: MMATG 203 and MMATG 204

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Use separate booklet for each course.

Course: MMATG 203(Topology II)
(Marks: 20)

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2= 20

1. (a) Let (X, τ) be a first countable space and $x \in X$ and let $\{W_i : i = 1,2,3 \ldots.\}$form a countable open base about x. Then prove that there exists an infinite subsequence $\{V_i : i = 1,2,3 \ldots.\}$ of the sequence $\{W_i : i = 1,2,3 \ldots.\}$ such that for any open set U containing x there exists an index m such that $V_i \subset U$ for all $i \geq m$. [5]

   (b) Prove that a first countable space is Hausdorff if every convergent sequence has unique limit. [5]

2. (a) Prove that every sequence in a countably compact space has a cluster point and hence show that a countably compact first countable space is sequentially compact. [5]

   (b) Is intersection of two compact subsets in a topological space compact? Support your answer. [5]

3. (a) Prove that a topological space (X, τ) is connected if and only if there does not exist any continuous function $f: X \to R$ such that $f(X)$ consists of exactly two points, the real number space R being equipped with usual topology. [5]

   (b) Prove that a topological space (X, τ) is locally connected if and only if each component of an open subspace is open in (X, τ). [5]

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Course: MMATG 204 (Differential Geometry-II)

(Marks: 20)

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2= 20

1. (a) Show that the normal curvature of any curve on a sphere of radius r is $\pm 1/r$. [5]

   (b) Compute the second fundamental form of the elliptic paraboloid $\sigma(u,v) = (u,\ v,\ u^2 + v^2)$. [5]

2. (a) Prove that a curve on a surface is a geodesic if and only if its geodesic curvature is zero everywhere. [5]

   (b) Prove that under isometry between two surfaces, geodesics remain invariant. [5]

3. (a) Prove that any map of any region of the earth's surface must distort distances. [5]

   (b) If $Edu^2 + 2Fdu\,dv + Gdv^2$ and $Ldu^2 + 2Mdu\,dv + Ndv^2$ are respectively the first and second fundamental form of a surface patch $\sigma(u,v)$, then prove that its Gaussian curvature is given by $\kappa = \frac{LN-M^2}{EG-F^2}$. [5]