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M.A./M.Sc. Semester II Examination, 2020 (CBCS)
Subject: Mathematics
Course: MMATG 201 and MMATG 202

Time: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Use separate booklet for each course.

Course: MMATG 201 (Real analysis - II)

(Marks: 20)

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2= 20

1. (a) Let E and F be two disjoint measurable subsets of $\mathbb{R}$ and $\varphi: E \cup F \to [0, \infty)$ be a measurable simple function. Prove that $(L) \int_{E \cup F} \varphi \; dx = (L) \int_E \varphi \; dx + (L) \int_F \varphi \; dx$. [5]

   (b) Let E be a measurable set, $f: E \to [0, \infty]$ be a measurable function and $\alpha > 0$ be a real number. Prove that $m(\{x \in E: f(x) > \alpha\}) \leq \frac{1}{\alpha} (L) \int_E f \; dx$. [5]

2. (a) Let E be a measurable set and $f, g : E \to \mathbb{R}^*$ be two measurable functions with $f = g \; a.e.$ on E, where $\mathbb{R}^* = \mathbb{R} \cup \{\infty, -\infty\}$. If f is Lebesgue integrable over E, then prove that g is Lebesgue integrable over E and $(L) \int_E f \; dx = (L) \int_E g \; dx$. [5]

   (b) With an example show that the point-wise limit of a sequence of Lebesgue integrable functions may not be Lebesgue integrable. [5]

3. (a) Let $\{f_n\}_{n \in \mathbb{N}}$ be a sequence of measurable functions from a measurable set E into $[0, \infty]$ converging point-wise to a function $f: E \to [0, \infty]$. Suppose further that $f_n \leq f$ on E, $\forall \; n \in \mathbb{N}$. Prove that $\lim_{n \to \infty} (L) \int_E f_n \; dx = (L) \int_E f \; dx$. [5]

   (b) Is the series $\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{(n)^{1/7}}, -\pi \leq x \leq \pi$, Fourier series of some bounded, Lebesgue integrable, 2π periodic function on $[-\pi, \pi]$? Support your answer. [5]

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Course: MMATG-202 (Complex Analysis - II)
(Marks: 20)

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2= 20

1. Let f be analytic in a domain D $(D \subset \mathbb{C})$. If at each point $z \in D$, $f'(z) \neq 0$, then show that the mapping $w = f(z)$ is conformal in D. [10]

2. (a) State and prove the Casorati-Weierstrass's Theorem. [1+4]

   (b) State and prove the Rouche's Theorem. [1+4]

3. Evaluate **any two** of the following integrals:

   (a) $\int_0^\infty \frac{\sin(mx)}{x} dx, \ m > 0.$ [5]

   (b) (PV) $\int_{-\infty}^{\infty} \frac{\cos(ax)}{x^2+m^2} dx$ for $a, m > 0$. [5]

   (c) $\int_0^\infty \frac{x^{1/2}}{x^3+1} dx.$ [5]