

Internal Assessment
M.A./M.Sc. Semester II Examination 2020 (for DDE)
Subject – Mathematics

Notation and symbols have their usual meanings

Time: 2 Hours

Full Marks: 25

MMATG-201 [Real Analysis - II]

Answer **any one** question. Only **first** answer will be evaluated.

1×5=5

1. Let $f : E \rightarrow \mathbb{R}^*$ be Lebesgue integrable on a measurable set E where \mathbb{R}^* is the set of extended real numbers. Show that $|f|$ is Lebesgue integrable on E and $|\int f dm| \leq \int |f| dm$ where $\int f dm$ and $\int |f| dm$ denote the Lebesgue integrals of f and $|f|$ over E respectively.
2. Let $f: [0,1] \rightarrow \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} \cos x, & x \text{ is rational in } [0,1] \\ \sin x, & x \text{ is irrational in } [0,1] \end{cases}$$

Show that f is Lebesgue integrable on $[0,1]$ but not Riemann integrable on $[0,1]$.

MMATG-202 [Complex Analysis - II]

Answer **any one** question. Only **first** answer will be evaluated.

1×5=5

1. If $g: G \rightarrow \mathbb{C}$ is analytic except at poles, show that the poles of f can not have a limit point in G , where G is a region.
2. Using Rouché's theorem, show that every polynomial equation over \mathbb{C} has n roots in \mathbb{C} .

MMATG-203 [Topology - II]

Answer **any one** question. Only **first** answer will be evaluated.

1×5=5

1. Show that in a first countable T_1 space, every singleton set is G_δ .
2. Show that in a countably compact space, every real-valued continuous function is bounded.

MMATG-204 [Differential Geometry - II]

Answer **any one** question. Only **first** answer will be evaluated.

1×5=5

1. Compute the second fundamental form of the elliptic paraboloid $\sigma(u, v) = (u, v, u^2, v^2)$. State Meusnier's Theorem. [3+2]
2. State Euler's Theorem. Show that every geodesic has constant speed. [2+3]

MMATG-205 [Calculus of \mathbb{R}^n - I]

Answer **any one** question. Only **first** answer will be evaluated.

1×5=5

1. When is a function $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ called differentiable at a point $a \in \mathbb{R}^n$? Find the derivative of the function $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ at any point $a \in \mathbb{R}^n$ if f is linear.

2. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be given by $f(x, y) = xy$, $(x, y) \in \mathbb{R}^2$. Then find $Df(5,6), (x, y) \in \mathbb{R}^2$.

MMATG-206 [Abstract Algebra-I]

Answer **any one** question. Only **first** answer will be evaluated.

1×5=5

1. Show that there exist 14 elements of order 3 in a non-commutative group of order 21.
2. Let G be a group of order 45. Show that G is commutative.

MMATG-207 [Operations Research]

Answer **any one** question. Only **first** answer will be evaluated.

1×10=10

1. Find the optimal EOQ for a product having annual demand 2400 units, ordering cost Rs. 100 per order, purchase cost Rs. 10 per unit for $0 \leq Q < 500$ and Rs. 9 per unit for $Q \geq 500$, and holding cost Rs. 20% of unit purchase cost.
2. Solve the LPP by revised Simplex method $\max z = -x_1 + 2x_2$, subject to $-x_1 + x_2 \leq 1$, $-x_1 + 2x_2 \leq 4$ and $x_1, x_2 \geq 0$.

MMATG-208 [Integral Transform]

Answer **any one** question. Only **first** answer will be evaluated.

1×5=5

1. Find the Fourier transform of $f(x) = \exp(-a|x|)$.
2. Find the Laplace transform of $f(t) = \frac{\sin at}{t}$.

MMATG-209 [Integral Equations]

Answer **any one** question. Only **first** answer will be evaluated.

1×5=5

1. Reduce the initial value problem $\frac{d^2y}{dx^2} + y = 0, y(0) = 0, (\frac{dy}{dx})_{at x=0} = 1$ into Volterra integral equations of second kind.
2. Solve $y(x) = \cos x + \frac{1}{2} \int_0^\pi \sin x y(t) dt$.

MMATG-209 [Numerical Methods]

Answer **any one** question. Only **first** answer will be evaluated.

1×5=5

1. What is the basic difference between ordinary interpolation and Hermite interpolation? Which one is more accurate and why?
2. Using the two point Gaussian quadrature rule, approximate the integral $I = \int_0^1 \frac{1}{1+x} dx$.