,

M.A./M.Sc. Semester II Examination, 2020 (CBCS)
Subject: Mathematics
Course: MMATG 205 and MMATG 206

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Use separate booklet for each course.

Course: MMATG 205 (Calculus of $\mathbb{R}^n$ -1)

(Marks: 20)

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2= 20

1. (a) State and prove Cantor's intersection theorem over $\mathbb{R}^n$. [2+5]

   (b) Let $A=[0,1]\times[0,1], B=\mathbb{R}^2$. Does there exist a continuous function $f:A\to B$ such that $f(A)=B$? Support your answer. [3]

2. (a) Let $f:\mathbb{R}^n\to\mathbb{R}^m$ be a function given by $f=(f_1, f_2,..., f_m)$. Let $c,u\in\mathbb{R}^n$. Then prove that $f'(c;u)$ exists if and only if $f_k'(c;u)$ exists for each k = 1,2, ..., m and $f'(c;u)=(f_1'(c;u), f_2'(c;u),..., f_m'(c;u))$. [5]

   (b) Let $f:\mathbb{R}^3\to\mathbb{R}^2$ be a function defined by $f(x,y,z)=(xyz, 2x+3y+4z), \forall(x,y,z)\in\mathbb{R}^3$. Find $Df(1,2,0)$. [5]

3. State and prove Mean value theorem for a function $f:\mathbb{R}^n\to\mathbb{R}^m$. Hence show that if $f:\mathbb{R}^n\to\mathbb{R}^m$ is differentiable and $Df(c)=\odot, c\in\mathbb{R}^n$, where $\odot$ is the zero function from $\mathbb{R}^n$ to $\mathbb{R}^m$ then f is constant on $\mathbb{R}^n$. [2+5+3]

,

Course: MMATG 206 (Abstract Algebra I)
(Marks: 20)

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2= 20

1. (a) Show that any group of order 6 is either S_3 or $\mathbb{Z}_6$ (upto isomorphism). [5]

   (b) Let G be a non-commutative group of order $2p$, p odd prime. Show that the order of the group of inner automorphisms of G is $2p$. [5]

2. (a) Let G be a finite group acting on a set X. Show that $|Gx| = [G\colon G_x]$, where Gx is the orbit of x in X and G_x, the stabilizer of x in G. [5]

   (b) Suppose $X = \{1,2,3\}$ is a S_3-set via the action $\sigma x = \sigma(x)$. Calculate $|S_3 x|$, for x =1,2,3. [5]

3. (a) Let G be a finite group of order p^n, where p is prime and $n \gneqq 0$. Then show that G has non-trivial centre $Z(G)$. [5]

   (b) Show that any group of order 2020 is not simple. [5]