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M.A./M.Sc. Semester II Examination, 2020 (CBCS)

Subject: Mathematics

Course: MMATG 208 and MMATG 209

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Use separate booklet for each course.

Course: MMATG 208 (Integral Transforms)

(Marks: 20)

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2= 20

1\. (a) Let f be a function of exponential order having finite discontinuity at some finite number of points in [0, ∞). Prove that $L\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0) - e^{-as}[f(a+0) - f(a-0)]$, where $F(s) = L[f(t)]$, the Laplace transform of f. [5]

(b) Prove that [5]

$L\left[\int_0^x \frac{e^t sint}{t}\,dt\right] = \frac{cot^{-1}(s-1)}{s}$, where $L[f(t)]$ is the Laplace transform of f.

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2\. (a) Determine the Fourier transform of $e^{-\frac{x^2}{2}}$. [5]

(b) Prove that [5]

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{\mathcal{F}}(f(x))\overline{\bar{\mathcal{F}}(g(x))}\,ds = \int_{-\infty}^{\infty} f(x)\overline{g(x)}\,dx.$$

3\. (a) Show that the conditions for a function $f(t)$ to possess Laplace transform is sufficient but not necessary. [5]

(b) Is $f(t) = e^{t^2}$ a function of exponential order? Justify your answer. [5]

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Course: MMATG 209 (Integral Equations)
(Marks: 20)

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2= 20

1. (a) Show that an integral equation of third kind can be converted into an integral equation of second kind with a modified kernel and unknown function under certain conditions to be stated by you. [3]

   (b) Convert the boundary value problem $\frac{d^2y}{dx^2} + xy = 1,\ y(0) = 0,\ y(1) = 1$ into an integral equation. [4]

   (c) Explain the concept of eigen values and eigen functions of a homogeneous Fredholm integral equation. [3]

2. Establish the existence and uniqueness of solution of Fredholm integral equation of second kind. [10]

3. (a) Solve the integral equation: $u(x) = x + \int_0^x (t-x)u(t)\,dt$ [4]

   (b) Solve the integral equation: [6]

$$\varphi(x) = x + \lambda \int_{-\pi}^{\pi} \left(x\cos\xi + \xi^2 \sin x + \cos x \sin\xi\right)\varphi(\xi)\,d\xi$$