Internal Assessment
M.A./ M.Sc. Semester-IV Examination,2019(DDE)
Subject: Mathematics (Pure Stream)(Old Pattern)

Use separate answer-sheet for each paper**(Answer of each paper should be limited to one A4 size page)**

Notations and symbols have their usual meanings

Time: 2 Hours   Full Marks: 20

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Paper :MPG 401
(Modern Algebra-III)

Answer any **one** question**.** Only **first** answer will be evaluated**.**   1×5 = 5

1. (a) State Fundamental theorem of Galois Theory.
   (b) Let K be a subfield of a field F. Then $G(F/K)$ is a subgroup of the group Aut$F$.   (2+3)
2. Let M be an R-module . Then the following are equivalent:
   (i) M is Artinian .
   (ii) M satisfies minimal condition for submodules .

Paper :MPG 402
Unit- I (General Topology-II)

Answer any **one** question**.** Only **first** answer will be evaluated**.**   1×3 = 3

1. Show that components of a totally disconnected space are it's points.
2. Show that the uniformity on a pseudo-metric space (X, d) has a countable base .

Unit- II (Functional Analysis-III)

Answer any **one** question**.** Only **first** answer will be evaluated**.**   1×2 = 2

1. If X is a non-trivial real normed linear space, then prove that the first conjugate space X^* of X is also non-trivial.
2. When is a sequence $\{x_n\}$ in a normed linear space X said to be weakly convergent to an element x of X ? If $x_n \to x$ weakly, then show that $\{\|x_n\|\}$ is bounded.   (1+1)

Paper :MPS 403
(Advanced Functional Analysis-II)
Special Paper-I

Answer any **one** question**.** Only **first** answer will be evaluated**.**   1×5=5

1. Let Y be a subspace of a normed linear space X with $\dim(Y) < \propto$. Then for each $x \in X$, there is a best approximation to x out of Y .

2. (a) If a commutative Banach algebra with identity e in which every non-zero member is invertible , then show that X is isometrically isomorphic to the scalar field $\mathbb{C}$.
   (b) Define a Gelfand topology.   (3+2)

Paper : MPS 403

(Differential Geometry of Manifolds-II)

Special Paper-I

Answer any **one** question**.** Only **first** answer will be evaluated. 1×5=5

1. (a) Define a manifold of constant curvature. Give an example of it.
   (b) When is a Riemannian manifold M said to be projectively flat ? (3+2)

2. (a) Prove that the product of two symmetric endomorphism of the tangent space $T_p M$ of a Riemannian manifold (M, g) at $p \in M$ is symmetric if and only if they commute with each other .
   (b) Define linear connection on a smooth manifold M . (3+2)

Paper :MPS 404

(Operator Theory and Application-II)

Special paper-II

Answer any **one** question**.** Only **first** answer will be evaluated**.** 1×5=5

1. If f is a bounded, symmetric sesquilinear functional , then show that $\|f\| = \|\hat{f}\|$.

2. (a) Let $T: X \to X$ be a compact linear operator on a normed linear space X . Then for every $\lambda \neq 0$ the null space $N(T_\lambda)$ of $T_\lambda = T - \lambda I$ is finite dimensional .
   (b) State spectral theorem for compact normal operators on a complex Hilbert space. (3+2)