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M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Applied Stream)

Paper: MAG 401 (Continuum Mechanics -III)

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **five** questions. Only **first five** answers will be evaluated. 9×5 = 45

1. (a) Define circulation. State and prove Kelvin's theorem on circulation. [2+5]

   (b) Show that the velocity components $u = a(x^2 - y^2), \upsilon = 2a(x^2 - xy), w = 0$ represent a possible liquid motion. [2]

2. (a) Define an inviscid fluid. Write down the stress tensor at a point of such fluid. [2+1]

   (b) Obtain Helmholtz's equation of vorticity for an incompressible inviscid fluid moving under conservative forces. [6]

3. (a) State and prove Milne-Thomson circle theorem for a two dimensional motion of an inviscid liquid. [5]

   (b) Show that in a two dimensional irrotational flow, velocity potential and stream function both satisfy Laplace equation. [4]

4. (a) Define a vortex filament. Prove that in a circulation preserving motion, vortex lines move with the fluid. [2+3]

   (b) Show that vortex lines cannot begin or end within the fluid. [4]

5. (a) Obtain the dimensionless form of Navier-Stokes' equations for steady motion of viscous incompressible fluid without body force. [5]

   (b) For steady motion of viscous incompressible fluid through a circular pipe under uniform axial pressure gradient, find the velocity distribution of the fluid. [4]

6. (a) Using Cisotti's equation find the complex potential for simple harmonic progressive wave. [5]

   (b) Show that the individual water particles underneath a progressive wave describe closed elliptic paths. [4]

7. (a) Obtain the expression for kinetic energy of stationary wave. [5]

   (b) Find the dissipation function for the flow given by $u = A(a^2 - y^2 - z^2)$, $\upsilon = 0$, $w = 0$, where u, υ, w are the components of velocity and A is a constant. [4]

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M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Pure Stream)

Paper: MPG401 (Modern Algebra - II)

Time: 2 Hours | Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **five** questions. Only **first five** answers will be evaluated. $9\times5 = 45$

1 (a) Let F/K be a finite field extension. Show that every element of F is algebraic over K. [5]

(b) Give an example with proper justification of an infinite extension F/K where each element of F is algebraic over K. [4]

2. Find through detailed computation of the following:

i. $[Q(\sqrt{7},\sqrt{3}): Q(\sqrt{3})]$ [6]

ii. $Basis\ of\ Q(\sqrt{7},\sqrt{3})$ over Q. [3]

3. (a) Show that π^2 and $\sqrt{\pi}$ are transcendental over the field of reals R. [3+3]

(b) Show that every non-constant polynomial over a field has a splitting field. [3]

4. (a) Find a splitting field S of $x^4 - 10x^2 + 21$ over Q. Find basis of S over Q. [4+2]

(b) If F is a finite field then show that F is not algebraically closed. [3]

5. (a) Show that every field of characteristic 0 is perfect. [5]

(b) Show that the Galois group of the polynomial $x^3 - 5$ over Q is isomorphic to S_3. [4]

6. (a) Let F/K be a field extension. Show that $Aut(F/K)$ permutes the roots of irreducible polynomials. [5]

(b) Show that $Aut(Q(\sqrt{2})/Q)$ is a cyclic group of order 2. [4]

7. Let R be a ring and let M be an R-module. Then show that M is Noetherian R-module if and only if every non-empty set of submodules of M contains a maximal element under inclusion if and only if every submodule of M is finitely generated. [3+3+3]