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M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Applied Stream)

Paper: MAG 402

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]
Write the answer to Questions of each Group in separate books.

Group - A (Elements of Quantum Mechanics)
(Marks: 27)

Answer any **three** questions. Only **first three** answers will be evaluated. 9×3 = 27

1 Show that in the scattering of electromagnetic radiation from a stationary electron the change in wave length of the radiation depends only on the angle of scattering. [9]

2 Define stationary state. Show that if the Hamiltonian is independent of time the Schrodinger equation admits a stationary state solution. [1+8]

3 Derive the equation of continuity of non-relativistic quantum mechanics. [9]

4 Show that solution of the Schrodinger equation corresponding to the one-dimensional harmonic oscillator reduces to the solution of Hermite differential equation. [9]

5 Calculate the ratio of the expectation value of potential energy to the expectation value of the kinetic energy of hydrogen atom in the ground state. [9]

Group - B (Chaos and Fractals)
(Marks: 18)

Answer any **two** questions. Only **first two** answers will be evaluated. 9×2 = 18

1 (a) Give the definition of chaotic map. Show that the doubling map $g : S \to S$ defined by $g(\theta) = 2\theta, \theta \in S$ is chaotic on the unit circle S. [2+3]

(b) Explain briefly 'boundary of chaos' and 'universality'. [4]

2 (a) Show that the logistic map with parameter $r = 4$ and the tent map are conjugate. [6]

(b) Define Lyapunov exponent for a map. [3]

3 (a) Prove that the length of von Koch curve is infinite. [3]

(b) How do you find the self-similar dimension of a fractal? Calculate similarity dimension of the Cantor set. [3+3]

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Group – A (General Topology- II)

(Marks: 27)

Answer any **three** questions. Only **first three** answers will be evaluated. 9×3 = 27

1 (a) Show that a topological space (X, τ) is connected if and only if there does not exist any continuous function $f: X \to R$ such that $f(X)$ consists of exactly two points, the real number space R being equipped with usual topology. [4]

(b) Is image of a locally connected space under a continuous map locally connected? Support your answer. [5]

2 (a) Prove that a topological space (X, τ) is locally connected if and only if each component of an open subspace is open in (X, τ). [5]

(b) Prove that a uniform space (X, μ) is T_1 if and only if the intersection of all members of μ is the diagonal in $X \times X$. [4]

3 If (X, τ) is a second countable T_4-space then prove that there exists a homeomorphism f of X onto a subspace of R_∞ where R_∞ is the infinite dimensional Euclidean space. [9]

4 What is meant by one point compactification of a topological space. State and prove Alexandroff's theorem on one point compactification. [9]

5 (a) Define uniform continuity between two uniform spaces. Show that a uniformly continuous function between two uniform spaces is continuous. [3]

(b) If X is a path connected space, then prove that for any pair of points x_0 and x_1 in X the fundamental groups $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are isomorphic. [6]

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Group - B (Functional Analysis - II)
(Marks: 18)

Answer any **two** questions. Only **first two** answers will be evaluated. $9 \times 2 = 18$

1 (a) Let X and Y be two normed linear spaces over the same field $F = (\mathbb{R}\ or\ \mathbb{C})$ and $T: X \to Y$ be a linear operator. Prove that T^{-1} exists and is continuous on its domain of definition if and only if there exists a constant $m > 0$ such that $m\|x\| \leq \|T(x)\|,\ \forall\, x \in X$. [5]

(b) Let S be a finite dimensional subspace of an inner product space X and x be any vector in X. Prove that there exists a unique vector $y_0 \in S$ such that $\|x - y_0\| \leq \|x - y\|,\ \forall y \in S$ and $(x - y_0) \perp S$. [4]

2 (a) Let M be a subspace of a real normed linear space X and let $f: M \to \mathbb{R}$ be a bounded linear functional. Prove that f can be extended to a bounded linear functional $F: X \to \mathbb{R}$ such that $\|f\| = \|F\|$. [5]

(b) Prove that every orthonormal set in a separable Hilbert space is countable. [4]

3 (a) When is a normed linear space said to be reflexive? If a normed linear space X is reflexive, then prove that $X^{/}$ is also reflexive, where $X^{/}$ is the conjugate space of X. [1+4]

(b) When is a sequence $\{x_n\}_{n\in\mathbb{N}}$ in a normed linear space X said to be weakly convergent to an element x of X? If a sequence $\{x_n\}_{n\in\mathbb{N}}$ in a normed linear space X is weakly convergent to $x \in X$, then prove that $\{\|x_n\|\}_{n\in\mathbb{N}}$ is bounded. [1+3]