M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Applied Stream)

Paper: MAS 403 (Viscous Flows, Boundary Layer Theory and MHD)

Time: 2 Hours | Full Marks: 45

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

Answer any **five** questions. Only **first five** answers will be evaluated. 9×5 = 45

1 What do you mean by MHD approximations? Find the expression for Lorentz force of a conducting material. Simplify this force using MHD approximations. [3+3+3]

2 (a) Establish $\frac{D}{Dt}\left(\frac{\vec{B}}{\rho}\right)=\left(\frac{\vec{B}}{\rho}\cdot\vec{\nabla}\right)\vec{\nabla}$, where symbols have their usual meaning. [6]

(b) Drive Lundquist criterion. [3]

3 Describe briefly the Hartman flow in MHD. Find the velocity components of the flow. [4+5]

4 State and prove Ferraro's iso-rotation theorem. [2+7]

5 What is pinch effect? Discuss linear pinch configuration and its stability behavior. [2+3+4]

6 Give the concept of magnetic force-free field. Show that a force-free field is always obtained as a superposition of two different fields which are to be characterized by you? [4+5]

7 Briefly discuss MHD Rayleigh problem. Find velocity components when kinematic fluid viscosity << magnetic diffusivity. [4+5]

M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Pure Stream)

Paper: MPS 403 (Advanced Functional Analysis II)

Time: 2 Hours | Full Marks: 45

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

Answer any **five** questions. Only **first five** answers will be evaluated. 9×5 = 45

1 Let X be a normed linear space and Y be a subspace of X such that $Dim(Y)<\infty$. Prove [5+4]

that there exists a best approximation to an element $x \in X$ out of Y. Does the result hold if Y is infinite dimensional? Justify your answer.

2 (a) If C is a non-empty closed convex subset of a Hilbert space H, then show that for each $x \in H$ there exists a unique best approximation from C to x. Hence show that C contains a unique vector of smallest norm. [6+3]

3 (a) Let B be a Banach algebra with identity e and let $b \in B$. Suppose $\mu \neq 0$ be a complex scalar such that $|\mu| > \|b\|$. Show that $(\mu e - b)^{-1}$ exists and $(\mu e - b)^{-1} = \sum_{k=1}^{\infty} \mu^{-n} b^{n-1} \left(b^0 = e\right)$. [5]

(b) Prove that the collection of all non-invertible elements in a Banach algebra with an identity element is a closed set in it. [4]

4 (a) If G be the collection of all invertible elements in a Banach algebra X with an identity element. Prove that the mapping: $G \to G$ given by $x \to x^{-1} (x \in G)$ is a homeomorphism. [5]

(b) Let B be a Banach algebra in which every non-zero element is invertible. Show that B is isometrically isomorphic to the complex field. [4]

5 (a) When is a normed linear space said to be reflexive? If X be a reflexive Banach space then prove that every bounded sequence in X^*, the first dual of X has a weakly convergent subsequence. [1+4]

(b) Find the first conjugate space of $\mathbb{R}^n$. Is $\mathbb{R}^n$ reflexive? Justify your answer [3+1]

6 (a) State and prove Uniform Boundedness Principle and hence show that space $\wp$ of all real valued polynomials $p(s) = \sum_{i=0}^{n} p_i s^i, s \in \mathbb{R}, p \in \wp$ with norm $\|p\| = \max_{0 \le k \le n} |p_k|$, index n is not fixed, is not complete. [4]

(b) Let X be a complex Banach algebra with an identity. If $x \in X$ and $\lambda \in \mathbb{C}$ then show that the resolvent function $x(\lambda)$ is analytic at every point of $\rho(x)$, the resolvent set of x. [5]

7 (a) Define weak* topology for a given normed linear space. If X is a normed linear space then prove that weak* topology on X^*, the first dual of X, is Hausdorff. [2+2]

(b) State and prove Gelfand Mazur Theorem. [5]

M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Pure Stream)

Paper: MPS 403 (Differential Geometry of Manifolds - II)

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **five** questions. Only **first five** answers will be evaluated. 9×5 = 45

1		State and prove fundamental theorem of Riemannian geometry.	[1+8]
2	(a)	Define a gradient vector field on a Riemannian manifold. If Z is a gradient vector field on a Riemannian manifold (M, g) with the Levi-Civita connection ∇ then prove that $g(\nabla_X Z, Y) = g(\nabla_Y Z, X)$ for any $X, Y \in \chi(M)$.	[2+3]
	(b)	Let (M, g) be a Riemannian manifold and p be a point of M. Prove that the product of two symmetric endomorphism of the tangent space T_pM is symmetric if and only if they commute each other.	[2+2]
3	(a)	Define almost complex manifold. Show that every almost complex manifold is of even dimensional.	[2+4]
	(b)	Show that, in an almost complex manifold M of dimension 2m, the almost complex structure F has m eigenvalues i and m eigenvalues -i.	[3]
4	(a)	Define Nijenhuis tensor on an almost complex manifold. Show that it is pure in both slots.	[2+3]
	(b)	Obtain a necessary and sufficient condition for a vector field in an almost complex manifold to be contravariant almost analytic.	[4]
5	(a)	Deduce Gauss and Weingarten formulae for the submanifold of a Riemannian manifold.	[6]
	(b)	Define a manifold of constant curvature with an example.	[3]
6	(a)	Define Einstein manifold. Prove that a Riemannian manifold (M^n, g) (of dimension n>2) of constant curvature is an Einstein manifold. Is the converse true? Justifying your answer.	[1+3+2]
	(b)	Show that the scalar curvature of an n (>2)-dimensional Einstein manifold is constant.	[3]
7	(a)	Define sectional curvature of a Riemannian manifold. Prove that the curvature tensor of a Riemannian manifold (M, g) at a point p is uniquely determined by sectional curvatures of all 2-dimensional tangent subspaces of T_pM at p.	[2+5]
	(b)	State Schur's Theorem.	[2]