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M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Applied Stream)

Paper: MAT 405 (Viscous Flows, Boundary Layer Theory and MHD)

Time: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **five** questions. Only **first five** answers will be evaluated. 10×5 = 50

1. Discuss Newton's law for fluid viscosity. Classify different kinds of non-Newtonian fluid models with examples. [4+6]

2. Write constitutive equations for MHD. Discuss interlocking phenomenon in MHD. Using MHD approximations simplify generalized Ampere law. [3+3+4]

3. The velocity distribution in the boundary layer over a plate is prescribed by $u = U\left(1-e^{-\eta}\right)$, $\eta = y/\delta$. Calculate the skin friction, displacement thickness, momentum thickness and energy thickness. [10]

4. Write short notes (any two) [10]
   (i) Stokes' paradox and Oscen correction for slow motion,
   (ii) 'Dynamo principle' in connection with Earth magnetic field,
   (iii) Self-similar flows,
   (iv) Magnetic Reynolds number and its significance.

5. State and prove Alfven's theorem. Give its physical significance. [8+2]

6. Give the mathematical formulation of MHD Couette flow. Obtain its velocity and induced magnetic field. [4+6]

7. (a) Show that a magnetic force-free field $\vec{H}$ satisfies the equation $\nabla^2\vec{H} + \alpha^2\vec{H} = 0$. Here α is a scalar quantity. [5]

   (b) Show that for a perfectly conducting fluid the constant pressure surface is a complete torus. [5]

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M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Applied Stream)

Paper: MAT 405 (Advanced Operations Research)

Time: 2 Hours Full Marks: 50

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[Notation and symbols have their usual meaning]

Answer any **five** questions. Only **first five** answers will be evaluated. 10×5 = 50

1. (a) State and prove the sufficient condition for optimality of Lagrange multiplier method for solving constrained optimization problem with equality constraints. [6]

   (b) Solve the following by constrained variation method:

   Minimize $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2$

   subject to $2x_1 + 4x_2 + 3x_3 = 9,\ 4x_1 + 8x_2 + 5x_3 = 17$ [4]

2. Solve the following quadratic programming problem using Beale's method

   Maximize $z = 2x_1 + 3x_2 - 2x_2^2$

   subject to the constraints $x_1 + 4x_2 \le 4,\ \ x_1, + x_2 \le 2$ and $x_1, x_2 \ge 0$ [10]

3. (a) Find the maximum flow in the network with the following arcs and arc capacities, flow in each arc being non-negative, v_a is the source and v_b, the sink.

Arc	(v_a, v_1)	(v_a, v_2)	(v_2, v_1)	(v_1, v_3)	(v_1, v_b)	(v_2, v_b)	(v_3, v_b)
Capacity	4	6	2	3	4	6	5

[5]

   (b) Using Fibonacci method

   Maximize $f(x) = \begin{cases} \dfrac{2x+3}{6}, & x \le 3 \\ 6 - x, & x > 3 \end{cases}$

   in the interval [-1,5] taking $n = 5$. [5]

4. (a) Define Fritz-John saddle point problem. [2]

   (b) State and prove Fritz-John saddle point necessary optimality theorem. When does the theorem fail? [7+1]

5. (a) Derive the steady state equations for the queueing model $(M/M/C\ :\ N/FCFS/\infty)$. [6]

   (b) Under what conditions this model reduces to other Poisson queueing models? [4]

6. (a) Using A.M.-G.M. inequality, reduce the unconstrained posynomial geometric programming problem into its dual form. [4]

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(b) Solve the following constrained optimization problem by geometric programming technique:

Minimize $z = \frac{4}{9} x_1^{-1} x_2 x_3^{-1/2} + x_1^{-1} x_3^{-1}$

subject to the constraint

$\frac{2}{9} x_1^{1/3} x_3 + 5 x_1 x_2^{-2/3} = 1, \quad x_i > 0, \quad i = 1, 2, 3$ [6]

7. (a) State Bellman's principle of optimality. [2]

(b) A man is engaged in buying and selling identical items. He operates from a warehouse that can hold 500 items. In each month, he can sell any quantity that he chooses up to the stock at the beginning of the month. He can also buy as much as he wishes for delivery at the end of each month so long as his stock does not exceed 500 items. For the next four months, he has the following forecasts of purchase costs and selling prices.

Month	:	1	2	3	4
Purchase cost	:	27	24	26	28
Selling price	:	28	25	25	27

If he has a current stock of 200 units, what quantities should he sell and buy in next four months? Solve the problem using dynamic programming technique. [8]

M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Pure Stream)

Paper: MPT 405 (Advanced Functional Analysis)

Time: 2 Hours Full Marks: 50

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Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

Answer any **five** questions. Only **first five** answers will be evaluated. 10×5 = 50

1 (a) Define vector lattice. Let X be a compact Hausdorff space. Examine if $C(X, \mathbb{R})$ is a lattice. [1+3]

(b) If a normed linear space is separable then prove that its first conjugate space is also separable. Does the converse hold? Justify your answer [4+2]

2 (a) If G be the collection of all invertible elements in a Banach algebra X with identity element e. If $x \in G$ and $h \in X$ such that $\|h\| < \frac{1}{2}\|x^{-1}\|^{-1}$ then show that $x + h \in G$ satisfying $\left\|(x+h)^{-1} - x^{-1} + x^{-1}hx^{-1}\right\| \le 2\|x^{-1}\|^{3}\|h\|^{2}$ [1+5]

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(b) Let X be a Banach Algebra with an identity e. If x is invertible and $y \in X$ satisfies $\|yx^{-1}\| < 1$ then show that $(x-y)$ is invertible and $(x-y)^{-1} = \sum_{j=0}^{\infty} x^{-1}\left(yx^{-1}\right)^{j}$ [4]

3 (a) Let P be an inner product space and Q be a subspace of P. Suppose $p \in P$ and $q \in Q$ such that $(p-q)$ is orthogonal to Q. Show that q is a best approximation from Q to p. [5]

(b) When is a normed linear space said to be strictly convex? If a normed linear space X is strictly convex then show that $\|x+y\| = \|x\| + \|y\|$, for all non zero vectors x and y in X implies $x=cy$, for some positive scalar c. [5]

4 (a) When is a normed linear space said to be reflexive? Find the first conjugate space of c_0. Is c_0 reflexive? Justify your answer. [1+3+2]

(b) Examine if $L_p[a,b](1 < p < \infty)$ is uniformly convex. [4]

5 (a) Prove that in a topological vector space T every convex neighbourhood of zero vector in T contains a balanced convex neighbourhood of zero vector of T. [5]

(b) State separation theorem in a topological vector space. Hence show that it is Hausdorff. [5]

6 (a) Let T be a topological vector space over the field of real numbers and let S be a subspace of T with $Dim(S) = k$. Prove that S is closed. [7]

(b) Let A be a convex balanced absorbing set containing zero vector of a topological vector space T and let q_A be the Minkowski functional of A over T. Show that $q_A(\gamma t) = |\gamma| q_A(t)$, for all scalars γ and for all $t \in T$. [3]

7 When is a topological vector space said to be normable? Prove that a topological vector space T is normable if and only if there is a convex bounded neighbourhood of zero vector in T [2+8]

M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Pure Stream)

Paper: MPT 405 (Differential Geometry of Manifolds)

Time: 2 Hours Full Marks: 50

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[Notation and symbols have their usual meaning]

Answer any **five** questions. Only **first five** answers will be evaluated. 10×5 = 50

1 (a) Define smooth manifold. [3]

(b) Prove that $\mathbb{R}P^n$ is an n-dimensional smooth manifold. [7]

2 (a) Define torsion tensor field T of an affine connection on a smooth manifold M. Show that T is $C^\infty(M)$-bilinear. [2+3]

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(b) Prove that an affine connection on a smooth manifold can be decomposed into a sum of a multiple of its torsion tensor and a torsion-free connection. [5]

3 (a) Define exterior derivative on a smooth manifold M. [2+3]

Show that $d(f\omega)=df\wedge\omega+fd\omega$ for any r-forms on a smooth manifold and

$f\in C^\infty(M)$, where 'd' denotes the exterior differentiation.

(b) If ω is an 1-form on a smooth manifold M of dimension n then prove that [5]

$$d\omega(X,Y)=\frac{1}{2}\{X\omega(Y)-Y\omega(X)-\omega([X,Y])\}$$

for any $X,Y\in\chi(M)$.

4 Write short notes on (i) vector bundle, (ii) tangent bundle and [3+4+3]

(iii) fundamental vector field.

5 (a) Define Lie algebra of the Lie group G. Prove that Lie algebra of the Lie group G is isomorphic to T_eG, $e\in G$ is the identity of G. [2+4]

(b) Deduce Maurer-Cartan structure equation of a Lie group G. [4]

6 (a) Define submanifold of a Riemannian manifold with an example. [2+2]

(b) Deduce Gauss and Codazzi equations for the submanifold of a Riemannian manifold. [6]

7 (a) Define an F-connection on an almost complex manifold. Obtain a necessary and sufficient condition for an affine connection on an almost complex manifold to be a F-connection. [2+3]

(b) When is F-connections on an almost complex manifold said to be H-projectively relative? Obtain a necessary and sufficient condition for two half symmetric F-connections to be H-projectively related. [2+3]

M.A./M.Sc. Semester IV Examination, 2019 (under DDE)

Subject: Mathematics (Pure Stream)

Paper: MPT 405 (Operator Theory and Applications)

Time: 2 Hours Full Marks: 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **five** questions. Only **first five** answers will be evaluated. 10×5 = 50

1 If X and Y are normed linear spaces and $T\in B(X,Y)$ then define conjugate operator T' of T. If X and Y are inner product spaces then define the adjoint T^* of an operator $T: D\to Y$ where D is a subspace of X. If X and Y are Hilbert spaces and $T\in B(X,Y)$ then find the [2+2+6]

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relation between $T^{'}$ and T^{*}.

2 Let X be a complex Hilbert space and let $A, B \in B(X, X)$ be such that $AB = BA$ and $A \geq 0, B \geq 0$. Prove that $AB \geq 0$. [10]

3 Let X be a normed linear space and Y be a Banach space and let $T \in B(X, Y)$ be compact. Prove that the conjugate $T^{'}$ of T is compact. [10]

4 (a) Give an example, with justification, of an operator which is continuous but not compact. [5]

(b) Let A, B: $X \rightarrow X$ be compact operators, X being a normed linear space. Show that $A+B$, AB, BA are compact operators. [5]

5 State and prove spectral theorem for compact normal operators on Hilbert spaces [10]

6 Let X be a complex Hilbert space and let $T \in B(X, X)$ be compact normal having finite point spectrum $\sigma_p(T) = \{\lambda_1, \lambda_2, \dots \dots .. \lambda_k\}$. Then prove that T is finite dimensional and $T = \sum_{i=1}^{k} \lambda_i E_i$ where E_i is the orthogonal projection on $N(T - \lambda_i I)$, $i = 1, 2, \dots . k$. [10]

7 State and prove spectral theorem for bounded self-adjoint operators on complex Hilbert spaces. [10]