,

M.A./M.Sc. Semester II Examination, 2019 (under DDE)

Subject: Mathematics

Paper: MCG 201

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Write the answer to Questions of each Group in separate books.

Group - A (Complex Analysis - II)

(Marks: 27)

Answer any **three** questions. Only **first three** answers will be evaluated. 9×3 = 27

1 (a) State and prove the Laurent's Theorem. [1+4]

(b) Prove that $e^{z+\frac{1}{z}} = \sum_{-\infty}^{\infty} a_n z^n$, where $a_n = \frac{1}{\pi}\int_0^{\pi} e^{2\cos\theta}\cos(n\theta)d\theta$ and $a_{n-1} - a_{n+1} = na_n$. [4]

2 (a) State and prove Riemann's Theorem for a removable singularity. [1+4]

(b) Evaluate $\int_0^{\infty} \frac{x^{\alpha-1}}{1+x^3}dx,\ 0<\alpha<3$ by the method of contour integration. [4]

3 (a) State and prove Argument Principle. Why is it called the Argument Principle? [1+3+2]

(b) Find the number of roots of $f(z) = z^{10} - 6z^9 - 3z + 1$ which lie interior to the unit circle $|z| = 1$. [3]

4 (a) Find the maximum modulus of the function $f(z) = z - 3i$ on $|z| \le 2$. [3]

(b) State and prove Schwarz lemma. [2+4]

5 Let $f(z)$ be analytic in a domain D ($D \subset \mathbb{C}$). If at each point $z \in D$, $f'(z) \neq 0$, then show that the mapping $w = f(z)$ is conformal in D. [9]

Group - B (Real Analysis - II)

(Marks: 18)

Answer any **two** questions. Only **first two** answers will be evaluated. 9×2 = 18

1 State and prove monotone convergence theorem. [2+7]

2 (a) Let $f: [0,1] \to \mathbb{R}$ be a function, defined by [4]

$$f(x) = \begin{cases} 7x, & \text{if } x \text{ is rational} \\ 9x^2, & \text{if } x \text{ is irrational} \end{cases}.$$

Evaluate $(L)\int_0^1 f\,dx$.

(b) Let E be a measurable subset of $\mathbb{R}$ and $f, g : E \to \mathbb{R}^*$ be two measurable functions such that $|f| \le g$ $a.e.$ on E, where $\mathbb{R}^* = \mathbb{R} \cup \{\infty, -\infty\}$. If g is Lebesgue integrable on E, then prove that f is Lebesgue integrable on E. [5]

,

3 Let $f(x) = \begin{cases} \pi + x, & \text{if } -\pi \leq x < 0, \\ \pi - x, & \text{if } 0 \leq x < \pi, \end{cases}$ and $f(x + 2\pi) = f(x)$. Find the Fourier series of f in $[-\pi, \pi]$ and hence find $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. [6+3]