

M.A./M.Sc. Semester II Examination, 2019 (under DDE)

Subject: Mathematics

Paper: MCG 202

Partial Differential Equations

Time: 2 Hours

Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Write the answer to Questions of each Group in separate books.

Group - B (Partial Differential Equations)

(Marks: 27)

Answer any **three** questions. Only **first three** answers will be evaluated.

9×3 = 27

- 1 (a) Eliminate the arbitrary function f from the equation $x + y + z = f(x^2 + y^2 + z^2)$. [4]
(b) Explain Cauchy's problem of first order partial differential equation. [5]
- 2 (a) Find the general solution of $y^2p - xyq = x(z - 2y)$. [4]
(b) Construct the general integral of the equation $(x - y)p + (y - x - z)q = z$ and particular solution through the circle $z = 1, x^2 + y^2 = 1$. [5]
- 3 (a) Classify the linear partial differential equation of second order. [3]
(b) Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form. [6]
- 4 (a) Discuss Charpit's method for solution of first order non-linear partial differential equation. Hence find the complete integral of $2zx - px^2 - 2qxy + pq = 0$. [5+4]
- 5 (a) Solve the Neumann problem for a circle. [5]
(b) Determine the solution of
$$U_{xx} - U_{yy} = 1$$
subject to
$$U(x, 0) = \sin x, U_y(x, 0) = x.$$
 [4]

Group - B (Differential Geometry)

(Marks: 18)

Answer any **two** questions. Only **first two** answers will be evaluated.

9×2 = 18

- 1 (a) Prove that the intrinsic derivative of an invariant coincides with its total derivative. [3]
(b) Deduce Serret-Frenet formulae for a space curve. [6]
- 2 (a) Obtain a necessary and sufficient condition for a vector field in E^3 to be parallel. [5]

- (b) Find the first fundamental form of the surface

[4]

$$x^1 = a \cos u^1 \sin u^2$$

$$x^2 = a \sin u^1 \sin u^2$$

$$x^3 = c \cos u^2$$

where a and c are constants.

- 3 (a) State and prove Meusnier's theorem.

[1+4]

- (b) Prove that a curve (regular) on a surface is a geodesic if and only if its geodesic curvature is zero everywhere.

[4]