Internal Assessment
M.A./ M.Sc. Semester-III Examination,2020(CDOE)
Subject: Mathematics (Pure Stream)(CBCS)

Answer of MMATP301 should be limited to one A4 size page,
Answer of MMATP302 & MMATP303 together should be limited to one A4 size page,
Answer of MMATG304 should be limited to one A4 size page,
Answer of MMATP305 should be limited to one A4 size page,
Answer of MMATPME306-1 should be limited to one A4 size page,
Answer of MMATPME307-6 should be limited to one A4 size page,
Answer of the minor elective MCSA-MIE303 should be limited to one A4 size page

Notations and symbols have their usual meanings

Time: 3 Hours **Full Marks: 60**

Paper :MMATP 301
(Abstract Algebra-II)

Answer any **one** question. Only **first** answer will be evaluated. $1\times10 = 10$

1. Let A and B be two submodules of an R-module M. Then show that $A+B$ is the smallest submodule of M containing $A \cup B$.
2. State and prove Gauss Lemma.

Paper :MMATP 302
(Functional Analysis-II)

Answer any **one** question. Only **first** answer will be evaluated. $1\times5 = 5$

1. State and prove Reisz Lemma.
2. Let X and Y be normed linear spaces over the same field of scalars. Let D be a subspace of X. Let $T: X \supset D \to Y$ be a closed linear transformation. Then show that if T^{-1} exists, it is also a closed transformation.

Paper:MMATP303
(Topological Vector Spaces)

Answer any **one** question. Only **first** answer will be evaluated. $1\times5 = 5$

1. a) Define Minkowski Functional.
   b) Let K be a convex absorbing set containing $\underline{0}$ of a Topological Vector Space X and let p_K be the Minkowski functional defined over X. Then show that $p_k(x) < 1 \Rightarrow x \in K$. **(2+3)**
2. Let X be a vector space $F=\mathbb{R}(\mathbb{C})$ and let M be a subspace of X. Then show that M is maximal if and only if $\dim \frac{X}{M} = 1$.

Paper :MMATG304
(Introduction to Manifolds)

Answer any **one** question. Only **first** answer will be evaluated. $1\times10=10$

1. a) Define the exterior algebra on a vector space.
   b) Obtain a necessary and sufficient condition for n numbers of 1-forms to be linearly independent. **(2+8)**
2. Define a Lie group. If a Lie group G has dimension n, then show that its Lie algebra $L(G)$ is also of dimension n. **(3+7)**

Answer any **one** question. Only **first** answer will be evaluated. $1\times5 = 5$

1. Let f be a sesquilinear functional and let $\hat{f}$ be the associated quadratic form. Then show that f is symmetric if and only if $\hat{f}$ is real valued.
2. Show that the annihilator of a subset and the orthogonal complement of a subset coincide in a Hilbert space.

Paper :MMATPME306-1
(Major Elective-1)
(Advanced Functional Analysis-I)

Answer any **one** question. Only **first** answer will be evaluated. $1\times10= 10$

1. Show that a locally convex space X is Bornologic if and only if every seminorm on X which is bounded on every bounded set, is continuous.
2. Show that the space $L_p[0,1]$ and $l_p(1 < p <\infty)$ are uniformly convex.

Paper :MMATPME307-6
(Major Elective-2)
(Operator Theory and Applications-I)

Answer any **one** question. Only **first** answer will be evaluated. $1\times10= 10$

1. Let X be complex Hilbert space and $A\in B(X)$. Then show that
   a) If A is normal, then $\overline{R(A)}=R(A^*)$.
   b) If A is normal, then $N(A)=N(A^2)$. (5+5)
2. Show that a compact operator $T: X \to Y$ maps weakly convergent sequence into a strongly convergent sequence.

Paper :MCSA-MIE303
(Minor Elective)
(Java for Scientific Computing)

Answer any **one** question. Only **first** answer will be evaluated. $1\times5= 5$

1. Explain the constructor method in Java.
2. What is an array in Java? How to initialize an array? (2+3)

Internal Assessment
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Paper :MMATA301
(Methods of Applied Mathematics)

Answer any **one** question. Only **first** answer will be evaluated.   1×5=5

1. Show that Dirac delta function is a singular generalized function.
2. Find the bilinear expansion of the Green's function for the system

$$-\frac{d^2u}{dx^2} - \mu x = f$$

Subject to the conditions *u(0)=u(1)=0.*

Paper :MMATA302
(Classical Mechanics-II)

Answer any **one** question. Only **first** answer will be evaluated.   1×5=5

1. Show that the transformation $Q = Log\left[\frac{1}{q}sinp\right], \dot{p} = qcotp$ is canonical.

2. a) What is generalised momentum?
   b) Define cyclic co-ordinate and give an example of it.   (2+2+1)

Paper :MMATA303
(Continuum Mechanics)

Answer any **one** question. Only **first** answer will be evaluated.   1×10=10

1. a) Determine Cauchy's stress quadric at *P* for a state of stress

$$(\tau_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

   b) Discuss the properties of Cauchy's stress quadric.   (6+4)
2. a) State the fundamental principles of continuum mechanics.
   b) Deduce the equation of continuity at a point of a continuous medium in the form

$$\frac{d\rho}{dt} + \rho\, div\, \vec{v} = 0.$$ (5+5)

Paper :MMATG304
(Theory of Electro Magnetic Fields and Relativity)

Answer any **one** question. Only **first** answer will be evaluated.   1×10=10

1. a) Write down Maxwell's equation on electrodynamics (in vacuum).Also mention the names of the symbols involved in the equation.
   b) Explain the concept of Retardad potential?   (3+2)+5

2. a) Define electric dipole. Hence obtain the electrostatic potential at an arbitrary point due to an electric dipole.
   b) Show that in free space each of the electric field vector and magnetic vector satisfies Wave equation. **(2+3)+5**

Answer any **one** question**.** Only **first** answer will be evaluated**.** 1×5=5

1. If the Dirichlet problem for a bounded region has a solution, then prove that it is unique.
2. State Maximum-Minimum principle for boundary value problem(for Dirichlet boundary condition).

Answer any **one** question. Only **first** answer will be evaluated. 1×10= 10

1. a) Define displacement and momentum thickness.
   b) Discuss the importance of boundary layer theory in fluid dynamics. **(5+5)**
2. a) Write down the significance of Reynolds number.
   b) Briefly explain Stoke's Paradox. **(5+5)**

Answer any **one** question. Only **first** answer will be evaluated. 1×10= 10

1. Describe Wolf's Modified Simplex Method for solving Quadratic Programming Problem.
2. Use the graphical method to minimizes the time needed to process the following jobs on the machines shown, i.e., for each machine find the job which should be done first. Also calculate the total elapsed time to complete both jobs.

	Machines				
Jobs1	Sequence: A	B	C	D	E
	Time: 3	4	2	6	2
Jobs2	Sequence: B	C	A	D	E
	Time: 5	4	3	2	6

Answer any **one** question. Only **first** answer will be evaluated. 1×5= 5

1. Explain the constructor method in Java.
2. What is an array in Java? How to initialize an array? **(2+3)**