M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATA305 (Boundary Value Problem)

Time: 1 Hour Full Marks: 20

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any ***two*** questions. Only ***first two*** answers will be evaluated. 2×10 = 20

(1) Find the Riemann Green function for the partial differential equation [10]

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{x+y} \qquad x, y > 0, xy > 1,$$

where the boundary conditions are

$$z = 0, \frac{\partial z}{\partial x} = \frac{2y}{x+y} \text{ at } xy = 1.$$

Hence find the solution.

(2) (a) Show that the solution of interior Neumann problem is not unique. [5]

(b) Write down and explain the conditions under which the series solution $u(x,t) = \sum_{n=1}^{\infty} U_n(x,t)$ of a hyperbolic partial differential equation will be convergent. [5]

(3) (a) Explain with examples: well-posed and ill-posed partial differential equations. [5]

(b) Show that the solution of the following mixed initial-boundary value problem is unique, [5]

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(0,t) = 0 = u(l,t) \qquad t \geq 0$$

$$u(x,0) = \varphi_0(x)$$

$$\frac{\partial u}{\partial t}(x,0) = \varphi_1(x) \qquad 0 \leq x \leq l.$$

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1 (a) Let X be a normed linear space. If S is a closed subspace of X, then prove that $a(S^a) = S$. [5]

(b) Let X and Y be two normed linear spaces. Prove that the linear transformation $T: X \to Y$ is closed if and only if its graph G_T is a closed subspace of $X \times Y$. [5]

2 (a) Let X be a complex Hilbert space and $A: X \to X$ be bounded linear and normal, then prove that $N(A) = N(A^2)$. [5]

(b) Let $\{T_n\}$ be a sequence of self-adjoint operators on a Hilbert space X such that $T_n \to T$ uniformly, then prove that T is also a self-adjoint operator. [5]

3 (a) Let f be a bounded sesquilinear functional on a Hilbert space X. Prove that [5]

$$\|f\| = \sup_{\|x\|=\|y\|=1} |f(x,y)|.$$

(b) Let f be a positive sesquilinear functional on the complex vector space X. Then prove that $|f(x,y)|^2 \leq \hat{f}(x)\hat{f}(y)$ for all $x, y \in X$. [5]