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M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATAME 306-1 (Boundary Layer Flows & Magnetohydrodynamics-I)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 4×10= 40

1. (a) Obtain the vector invariant form of Navier-Stokes' equations for viscous incompressible fluid. [5]

   (b) Discuss Stokes' second problem and formulate the problem mathematically. [2+3]

2. The approximate velocity distribution in the boundary layer flow over a flat plate is given by $u = U(1-e^{-\eta}), \eta = \dfrac{y}{\delta}$. Calculate wall skin-friction and displacement thickness [5+5]

   Here, U is the main stream velocity, η is the similarity variable, δ denotes the boundary layer thickness.

3. Briefly discuss the Blasious flow problem. Formulate the problem mathematically and obtain the self-similar equation with the appropriate boundary conditions for this problem. [2+3+5]

4. Write short notes on any two of the following: [5+5]

   (i) Boundary layer thickness

   (ii) Stokes' paradox

   (iii) Two layer theory of turbulence

5. Describe the steady boundary layer flow of a viscous fluid in a converging channel between two plane walls. Derive the self-similar equation for such flow. Also, obtain the velocity components for this flow. [2+4+4]

6. Explain 'closure problem' in turbulence. What is meant by 'mixing length theory'? Describe, in detail, the factors that affect the transition from laminar to turbulence. [3+2+5]

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M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATAME 306-2 (Turbulent Flows - I)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

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[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 4×10= 40

1. State Stokes' hypothesis for an isotropic fluid continuum. Establish Stokes' law of friction for moving fluid. [4+6]

2. Write the basic principles for boundary layer flows. Obtain the two-dimensional incompressible Prandtl boundary layer equations for flow past a plate. [2+8]

3. Discuss briefly boundary layer flow structure due to spread of a two-dimensional jet. Set up equations of this jet flow. Determine self-similar velocity components of this jet flow. [3+3+4]

4. Why do we need statistical description of turbulence? Explain briefly temporal, spatial and ensemble averages for turbulent velocity components. Show that the mean of fluctuating velocity components are zero. [3+4+3]

5. Write Reynolds' decomposition principle for flow variables in turbulence. Using this principle, establish the Reynolds' Average Navier-Stokes' equations. [2+8]

6. Describe three following important zones in turbulent flow near a wall: [6+4]

    (i) laminar sub-layer zone,
    (ii) buffer zone,
    (iii) turbulent zone.

    Establish the logarithmic velocity distribution in the turbulent zone.

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M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATAME 306-3 (Space Sciences-I)

Time: 2 Hours Full Marks: 40

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[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 4×10= 40

1. Show that the equation $\frac{\partial \vec{B}}{\partial x} = -\frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$ is not invariant under Galilean transformation. [10]

2. For Michelson Morley's experiment, show that [10]
time required to travel the vertical direction = $\frac{2L}{c}(1+\frac{v^2}{2c^2})$,
where L is the length of the interferometer arm.

3. (a) Show that the quantity $S^2 = c^2t^2 - x^2$ is a Lorentz invariant. [5]
(b) Explain the concept of light cone. [5]

4. Show that a time like vector cannot be orthogonal to another time like vector. [10]

5. (a) Show that moon feels much attractive force by the sun than earth. [5]
(b) What is a main sequence star? Explain briefly. [5]

6. (a) Describe different time spans of different elements burning at the core of a massive star. [5]
(b) When does a neutron star form a pulsar? [5]

M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATPME 306-1 (Advanced Functional Analysis-I)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
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[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 4×10= 40

1 State and prove Krein Milman Theorem. [2+8]

2 (a) Define a local base in a topological vector space. Prove that every topological vector space has a balanced local base. [1+4]
(b) Define a strictly convex normed linear space. If X is a strictly convex normed linear space such that for $x, y \in X$, $\|x+y\| = \|x\| + \|y\|$ holds then show that $x = cy$ for some scalar c. [2+3]

3 (a) Define a locally convex topological vector space. Is any topological vector space a locally convex topological vector space? Justify your answer. [2+3]

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(b) Prove that in a locally convex topological vector space X, the balanced, closed convex neighbourhoods of θ, the zero vector in X form a neighbourhood base of θ in X (supporting results may be assumed). [5]

4 State and prove Kolmogorov theorem. [2+8]

5 (a) When is a topological vector space said to be complete? Prove that a complete subset of a Hausdorff topological vector space is closed. [2+3]

(b) When is a filter said to be convergent to a point in a topological vector space? Show that a closed subset of a complete topological vector space is complete. [2+3]

6 Define a Bornological space. Obtain a necessary and sufficient condition for a locally convex topological vector space to be Bornological? [2+8]

M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATPME 306-4 (Rings of Continuous Functions - I)

Time: 2 Hours Full Marks: 40

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[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 4×10= 40

1 (a) Show that every zero set is a G_δ set. Show that in a Tychonoff space, every compact G_δ set is a zero set. [2+3]

(b) Show that for $f \in C(X)$, there exists a unit $u \in C^*(X)$ such that $fu \in C^*(X)$. [5]

2 Show that a space is pseudocompact if and only if $f[X]$ is compact for every $f \in C^*(X)$. [10]

3 (a) If X is basically disconnected, then show that for every $f \in C$, there exists a unit u of C such that $f = u|f|$. [5]

(b) Show that for any $f \in C(X)$, $f^{\frac{1}{3}} \notin (f)$, for any infinite space X. [5]

4 Show that a z-ideal I is prime in $C(X)$ if and only if every member of $C(X)$ does not change sign on some member of $Z[I]$. [10]

5 (a) Show that in $C(X)$, if the ideal generated by $f, g \in C(X)$ is a z-ideal, then it is a principal ideal. [5]

(b) Let X be a Tychonoff space. Let $\{V_\alpha\}$ be a family of disjoint sets in X with non-empty interiors, and such that for each index α, the set $\bigcup_{\sigma\neq\alpha} V_\sigma$ is closed. Show that any set D formed by selecting one element from the interior of each V_α is c-embedded in X. [5]

6 Show that a space X is a $p-$ space if and only if $C(X)$ is a regular ring. [8+2]

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M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATPME 306-5 (Advanced Complex Analysis - I)

Time: 2 Hours Full Marks: 40

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Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 4×10= 40

1 Find the abscissa of convergence C and the abscissa of absolute convergence A of the following Dirichlet series:

a) $\sum_{n=0}^{\infty} e^{-n^3} e^{-zn^2}$, [5+5]

b) $\sum_{n=1}^{\infty} \frac{1}{n^2} e^{-z \ln n}$.

2 State and prove the theorem of Borel and Caratheodary. [2+8]

3 Derive the formula for the type σ of an entire function f of fine order ρ in terms of Taylor's coefficients of $f(z)$. [10]

4 Suppose that $z_1, z_2, \cdots, z_m$ are distinct A_0-points of an entire function f of order ρ, where z_j is of order k_j $(j = 1,2,...,m)$. Prove that ρ is an integer and $f(z)$ is of the form $f(z) = A_0 + (z - z_1)^{k_1} (z - z_2)^{k_2} \cdots (z - z_m)^{k_m} e^{P(z)}$, where $P(z)$ is a polynomial of degree ρ. [10]

5 Find an order ρ and the type σ of the following entire functions:

a) $e^z \cos z$; [5+5]

b) $z^2 e^{2z} - e^{3z}$.

6 State and prove Hadamard's Factorization theorem. [2+8]

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M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATPME306-6 (Measure and Integration-I)

Time: 2 Hours Full Marks: 40

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[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 4×10= 40

1 (a) Give an example with proper justification of an algebra which is not a σ-algebra. [4]

(b) Let E be a countable class of subsets of $X\ (\neq \emptyset)$. Prove that $R(E)$ is countable, where $R(E)$ is the ring generated by E. [6]

2 (a) Let μ be a measure on a ring R of subsets of $X\ (\neq \emptyset)$ and $E \in R$. If $E_1, E_2, \ldots, E_n \in R$ and $E \subset \bigcup_{i=1}^{n} E_i$, then prove that $\mu(E) \leq \sum_{i=1}^{n} \mu(E_i)$. [3]

(b) Let μ be a finite, non-negative, additive set function defined on a ring R of subsets of $X\ (\neq \emptyset)$. If μ is either continuous from below at every $E \in R$ or continuous from above at $\emptyset$, then prove that μ is a measure on R. [7]

3 (a) Let μ be a measure on a ring R and $\overline{S}$ be the class of all μ^* measurable sets in $H(R)$. Prove that $S(R) \subset \overline{S}$, where μ^* is the outer measure on the hereditary σ-ring $H(R)$ induced by the measure μ and $S(R)$ is the σ-ring generated by R. [5]

(b) Let (X, S, μ) be a measure space and $f: X \to [0, \infty)$ be a measurable simple function. If $E, F \in S$ and $E \cap F = \emptyset$, then prove that $\int_{E \cup F} f\, d\mu = \int_E f\, d\mu + \int_F f\, d\mu$. [5]

4 Let (X, S, μ) be a finite measure space and $f_n\ (n = 1, 2, \ldots), f$ be real valued measurable functions defined on X such that $f_n \to f\ a.e.$. Prove that for any $\delta > 0$, there exists a measurable set F such that $\mu(F) \leq \delta$ and $\{f_n\}_{n \in \mathbb{N}}$ converges to f uniformly on $X - F$. [10]

5 (a) Let (X, S) be a measurable space, E be a measurable subset of X and for each $n \in \mathbb{N}$, $f_n: E \to \mathbb{R}^*$ be a measurable function. Prove that $\sup_n f_n$ is measurable. (Here $\mathbb{R}^* = \mathbb{R} \cup \{\infty, -\infty\}$). [3]

(b) Let (X, S, μ) be a measure space, $f_n\ (n = 1, 2, \ldots)$, $g_n\ (n = 1, 2, \ldots)$, f be real valued measurable functions defined on X such that $f_n \to f$ *in measure* and $f_n = g_n\ a.e.$ on X. Prove that $g_n \to f$ *in measure*. [4]

(c) Let (X, S) be a measurable space, E be a measurable subset of X and $f: E \to \mathbb{R}^*$ be a measurable function. Show that $|f|$ is measurable. [3]

6 (a) Let (X, S, μ) be a measure space and $\varphi,\ \psi : X \to [0, \infty]$ be measurable functions such that $\varphi \leq \psi$. Prove that for each $E \in S$, $\int_E \varphi\, d\mu \leq \int_E \psi\, d\mu$. [5]

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(b) Let (X, S, μ) be a measure space and $f: X \to \mathbb{R}^*$ be a measurable function on X. If f is integrable on X, then prove that f is finite valued $a.e.$ on X. [5]