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M.A./M.Sc. Semester III Examination, 2020 (CBCS)
Subject: Mathematics [New Syllabus]
Course: MMATAME307-1(Advanced Optimization-I)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. $4\times10 = 40$

1. (a) Show that the Kuhn Tucker necessary conditions for the optimization problem [4]

Minimize $f(x)$

subject to $g_i(x) \le 0, \quad i = 1, 2, \cdots, m$

and $h_j(x) = 0, \quad j = 1, 2, \cdots, l$

are also sufficient conditions if $f(x)$ is convex and $g_i(x)$, $i = 1, 2, \cdots, m$ are convex functions of x and $h_j(x)$ are linear.

(b) Use the Kuhn-Tucker necessary conditions to solve the following optimization problem: [6]

Maximize $z = 2x_1 - x_1^2 + x_2$

subject to $2x_1 + 3x_2 \le 6$, $2x_1 + x_2 \le 4$ and $3x_1 + 9x_2 = 16$.

2. (a) Write down Wolfe's algorithm for solving quadratic programming problem. [4]

(b) Use Beale's method for solving the quadratic programming problem: [6]

Maximize $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

subject to $x_1 + 2x_2 \le 2$ and $x_1, x_2 \ge 0$.

3. (a) If the iterative sequence $\left\{x^{(k)}\right\}$ be defined as [5]

$$x^{(k+1)} = x^{(k)} + \lambda^{(k)} d^{(k)}$$

where $d^{(k)}$ is given by

$$d^{(k)} = -M_k \nabla f\left(x^{(k)}\right)$$

and also, if $\nabla f\left(x^{(k)}\right) \ne 0$ and M_k is positive definite, then show that the iterative procedure possesses decent property.

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(b) Using steepest descent method, minimize $f(x) = x_1^2 + x_2^2 + 2gx_1 + 2fx_2 + c$ starting from the point $\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$. [5]

4. (a) For the quadratic function $f(x) = \frac{1}{2}\langle Ax, x\rangle$ (A being positive definite), show that the gradient vectors $\left\{g^{(k)}\right\}$ are mutually orthogonal and the direction search vectors $\left\{d^{(k)}\right\}$ are mutually A-conjugate. [5]

(b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ starting from $x^{(0)} = \begin{pmatrix}0\\0\end{pmatrix}$ using conjugate gradient method. [5]

5. (a) Discuss Fibonacci search method for solving one-dimensional non-linear minimization problem. [4]

(b) Using Fibonacci search method [6]

Maximize $f(x) = \begin{cases} \dfrac{2x+3}{6} & \text{for } x \le 3 \\ -x+6 & \text{for } x > 3 \end{cases}$

in the interval $[-1, 5]$ (consider 6 experimental points).

6. (a) Define Slater's, Karlin's and strict constraint qualifications. [3]

(b) Show that [3+4]

(i) Slater's constraint qualification and Karlin's constraint qualification are equivalent.

(ii) The strict constraint qualification implies Slater's constraint qualification and Kerlin's constraint qualification.

M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATAME307-2 (Advanced Operations Research-I)

Time: 2 Hours Full Marks: 40

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Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 4×10 = 40

1 Let the probability density function of a certain item during a day be [10]

$$f(x) = \begin{cases} 0.02 - 0.0002x, 0 \le x \le 100 \\ 0, x > 100 \end{cases}.$$

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The demand is assumed to occur with uniform pattern during the whole day. Let the unit carrying cost of the item in inventory be Rs. 0.50 per day, unit shortage cost be Rs. 4.5 per day, and the purchasing cost per unit be Rs. 0.50. Formulate the probabilistic inventory model and then determine the optimum stock level and corresponding total expected cost.

2 (a) State Johnson's algorithm for processing n jobs through 3 machines in sequencing problem. [3]

(b) There are 5 jobs, each of which must go through the three machines M_1, M_2 and M_3 in the order $M_1 M_2 M_3$. Processing time (in hours) are given in the following table: [2+3+2]

Job	J_1	J_2	J_3	J_4	J_5
Machine M_1	4	9	8	6	5
Machine M_2	5	6	2	3	4
Machine M_3	8	10	6	7	11

Passing is not allowed. Find the optimal sequence(s) of jobs that minimize(s) the total elapsed time to complete the jobs. Find the minimum elapsed time and the idle time of each machine.

3 (a) Show that the entropy function is maximum when mutually exclusive events are equal probable. [4]

(b) Let X and Y be two discrete random variables, each taking a finite number of values. Show that $H(X,Y) = H(Y|X) + H(X)$ and $H(Y) \geq H(Y|X)$, where $H(X,Y)$ is the joint entropy and $H(Y|X)$ is the conditional entropy function. [4+2]

4 Using KKT conditions to solve the following problem: [10]

Minimize $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 + 30\,x_1 + 20\,x_2 + 350$

subject to $x_1 - 60 \geq 0, x_1 + x_2 - 120 \geq 0, x_1 + x_2 + x_3 - 180 \geq 0$.

5 (a) Write down Beal's algorithm for solving quadratic programming problem. [3]

(b) Apply Wolf's method to solve the following quadratic programming problem [7]

Minimize $z = 2x_1^2 + 2x_2^2 + 2x_1x_2 - 4\,x_1 - 6\,x_2$

subject to $x_1 + 2x_2 \leq 0, x_1, x_2 \geq 0$.

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6 (a) A small project is having six activities. The relevant data about these activities is given in the following table: [(1+2)+2+5]

Activity	Time (days)		Cost (Rs.)	
	Normal	Crash	Normal	Crash
(1,2)	8	6	100	200
(1,3)	4	2	150	350
(2,4)	2	1	50	90
(2,5)	10	5	100	400
(3,4)	5	1	100	200
(4,5)	3	1	80	100

The indirect cost per day is Rs. 70.

i. Draw the project network and find the critical path.

ii. What is the normal project duration and associated cost?

iii. Crashing systematically the activities, determine the optimal project duration and the corresponding cost.

M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATPME307-3 (Euclidean and non-Euclidean Geometries-I)

Time: 2 Hours Full Marks: 40

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[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. $4\times10 = 40$

1. (a) State Circle-Circle, line-Circle and segment-Circle continuity principles. [5]

   (b) Give an example of a plane to show that segment–Circle continuity principle does not hold. [5]

2. State thirteen postulates of Hilbert plane and also Hilbert-Euclidean parallel postulate. [10]

3. (a) What do you mean by a non-Archimedean field? Give an example of a non-Archimedean ordered field. [5]

   (b) Define Euclidean plane, Pythagorean plane and hyperbolic plane. [5]

4. State and prove Desargues Theorem in an affine plane of order 2 over a finite filed. [10]

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5. (a) Prove that a linear isometry on a real Euclidean plane is either rotation by an angle or reflection about a straight line through the origin. [6]

(b) Deduce the curvature of a hyperbola. [4]

6. State and prove Fano's Theorem in projective plane of order 2 over a finite field. [10]

M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATPME307-5 (Advanced Differential Geometry - I)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 4×10 = 40

1 (a) Define a linear connection on a smooth manifold M. Derive the local expression of it. [3+4]

(b) Write the differences between a covariant derivative and a Lie derivative. [3]

2 (a) When is a linear connection said to be torsion free? Find the necessary and sufficient condition for a linear connection to be torsion free. [1+4]

(b) Define the curvature tensor R of a smooth manifold M. Also prove that $R(fX+gY,Z)U = fR(X,Z)U + gR(Y,Z)U$ for any $X,Y,Z,U \in \chi(M)$ and $f,g \in C^{\infty}(M)$. [2+3]

3 (a) Define a Riemannian manifold. Give an example of a Riemannian manifold. [2+2]

(b) Prove that if a Riemannian manifold $(M^n$,g) of dimension $n > 3$ admits a semi-symmetric metric connection whose Ricci tensor vanishes then the Riemannian curvature tensor of the semi-symmetric metric connection is equal to the Weyl conformal curvature tensor of the manifold. [6]

4 (a) State Schur's Theorem. [2]

(b) Derive equations of Gauss and Codazzi for a submanifold of a Riemannian manifold. [8]

5 (a) When is a Riemannian manifold $(M^n$,g) said to be an Einstein manifold? Give an example of an Einstein manifold. [1+1]

(b) Prove that every 3-dimensional Einstein manifold is a manifold of constant curvature. [8]

6 (a) When is a vector field on a Riemannian manifold called gradient vector field? [2]

(b) If Z is a gradient vector field on a Riemannian manifold $(M^n$,g) then prove that $g(\nabla_X Z, Y) = g(\nabla_Y Z, X)$ for any $X, Y \in \chi(M)$. [4]

(c) Prove that a Riemannian manifold $(M^n$,g) of dimension $n > 3$ is of constant curvature if and only if it is concircularly flat. [4]

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M.A./M.Sc. Semester III Examination, 2020 (CBCS)
Subject: Mathematics [New Syllabus]
Course: MMATAME307-6 (Quantum Mechanics-I)

Time: 2 Hours — Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. $4\times10 = 40$

1. For the one dimensional motion of a particle, determine the form of the wave function in the configuration space for which the product of the uncertainty in position and momentum of the particle is the minimum. [10]
2. Write down the quantum mechanical Hamiltonian H of the one dimensional harmonic oscillator. Determine a non-trivial operator A such that A commutes with H. Hence determine the matrix elements H relative to the eigen basis of A. [2+5+3]
3. Show that the infinitesimal translation of time in a quantum mechanical system is implemented by an infinitesimal operator. Determine the generator of that operator. Hence determine the operator which implements finite translation of time. [8+1+1]
4. Let a system consisting of two independent particles having angular momentum eigen values 1/2 of each. Find the eigen states of the total angular momentum operators. [10]
5. Determine the ground state energy of the normal helium atom using perturbation method (upto first order correction). [10]
6. Show that the Dirac equation describing the relativistic motion of particle automatically incorporates the spin of the particle. [10]

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M.A./M.Sc. Semester III Examination, 2020 (CBCS)

Subject: Mathematics [New Syllabus]

Course: MMATPME307-6 (Operator Theory and Applications I)

Time: 2 Hours — Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. $4 \times 10 = 40$

1 (a) Let X and Y be two normed linear spaces. When is a linear transformation $A: X \to Y$ said to be compact? Prove that A is compact if and only if it maps every bounded sequences $\{x_n\}$ in X onto a sequence $\{Ax_n\}$ in Y which has a convergent subsequence. Hence or otherwise show that if $A, B: X \to Y$ are compact operators then $A+B$ is compact. [6]

(b) Prove that a compact operator $T: X \to Y$ maps a weakly convergent sequence into a strongly convergent sequence, X, Y being normed linear spaces. [4]

2 (a) Let H_1 and H_2 be two complex Hilbert spaces and $T: H_1 \to H_2$ be a continuous linear operator such that T^*T is compact. Then prove that T is compact. [5

(b) Let $T: l_2 \to l_2$ be defined by $Tx = y$ where $x = \{\xi_1, \xi_2, \dots, \xi_n, \dots\}, y = \{\frac{\xi_1}{1}, \frac{\xi_2}{2}, \dots, \frac{\xi_n}{n}, \dots\}$. Show that T is compact. [5]

3 (a) If X is a finite-dimensional normed linear space and $T: X \to X$ is a linear operator then show that any spectral value of T is an eigen value of T. If X is infinite dimensional then give an example with justification that T can have spectral values which are not eigen values. [7]

(b) Let X be a normed linear space and $T: X \supset D(T) \to X$ be linear. Show that [3]

$\sigma_p(T) \cup \sigma_c(T) \subset \pi(T)$.

4 Let $T: X \to X$ be bounded linear where X is a complex Banach space. Prove that $\sigma(T)$ is compact. [10]

5 (a) Let X be a complex Hilbert space and $T \in B(X, X)$ be normal. Prove that $\sigma_r(T) = \varphi$. [5]

(b) Let $T: X \to X$ be a compact linear operator on a normed linear space X. Then prove that any nonzero approximate proper value of T is an eigen value of T. [5]

6 Prove that the set of all eigen values of a compact linear operator $T: X \to X$ on a normed linear space X is countable and the only possible point of accumulation is 0. [10]