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M.A./M.Sc. Semester III Examination, 2020 (CBCS)
Subject: Mathematics [New Syllabus]
Course: MMATG304 (Theory of Electro Magnetic Fields and Relativity)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 10×4 = 40

1. Derive the expression of energy stored in a magnatostatic field in vacuum. [10]
2. Show that solving the Maxwell's equations in vacuum is equivalent to solving an inhomogeneous wave equation with a source term. [10]
3. Show that at the surface of jump discontinuity of the electrostatic field vector $\boldsymbol{E}$, tangential component of $\boldsymbol{E}$ is continuous. [10]
4. Show that the potential at a point due to an electrically polarized substance is same as we should obtain if we were to suppose that there was a volume distribution of charge of density D_1 throughout the dielectric, and a surface charge distribution of density D_2 on the boundary of the substance, where D_1 and D_2 have to be defined by you. [10]
5. (a) Derive the Lorentz transformations. [8]
   (b) Why do we consider linear transformations to construct Lorentz transformations? [2]
6. (a) Prove that any four vector orthogonal to a space like vector may be a time like, light like or space like vector. [5]
   (b) Show that norm of any four vector is equal to c^2, where c is the speed of light. [5]

M. Sc. Semester III Examination, 2020 (CBCS)
Subject: Mathematics [New Syllabus]
Course: MMATG304 (Introduction to Manifolds)

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning.]

Answer any ***four*** questions. Only ***first four*** answers will be evaluated. 10× 4 = 40

1 (a) Define a smooth manifold of dimension n. [3]
  (b) Show that the n-sphere $S^n = \{(x^1, x^2, \cdots, x^{n+1}) \in \mathbb{R}^{n+1} : \sum_{i=1}^{n+1} (x^i)^2 = 1\}$ is an n-dimensional smooth manifold. [7]

2 (a) Define push forward map and pullback map of a smooth map f between two smooth manifolds. Also write the relation between them. [2+2+2]

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(b) Let $f: M \to N$ be a smooth map where *M* and *N* are two smooth manifolds of dimension *m* and *n* respectively. Prove that $f^*(gw) = (gof)f^*w$ for any $g \in C^\infty(N)$ and $w \in T^*N$. [4]

3 (a) Let *M* be a smooth manifold. Show that $[fX, gY] = fg[X,Y] + f(Xg)Y - g(Yf)X$ for any $X, Y \in \chi(M), f, g \in C^\infty(M)$. [3]

(b) Let *M* and *N* be two smooth manifolds and $f: M \to N$ be a smooth map. If X_i and Y_i, $i = 1, 2$ are f-related vector fields on *M* and *N* respectively then show that $[X_1, X_2]$ and $[Y_1, Y_2]$ are f-related. [4]

(c) Compute [*X*, *Y*], where $X = x^1x^2 \frac{\partial}{\partial x^1}$ and $Y = x^2 \frac{\partial}{\partial x^2}$. [3]

4 (a) Define a distribution on a smooth manifold. When is a distribution said to be involutive. [2+1]

(b) Show that every integrable distribution is involutive. [3]

(c) Find the integral curves of the vector field $X = y\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 2\frac{\partial}{\partial z}$ on $\mathbb{R}^3$. [4]

5 (a) Obtain a necessary and sufficient condition so that the 1-forms $\omega_1, \omega_2, \cdots, \omega_r$ are linearly dependent. [5]

(b) Compute the exterior product $(7du^1 + 4du^2)\wedge(3du^1 + 2du^2)$. [2]

(c) Let *M* be a smooth *n*-manifold and $f \in C^\infty(M)$. Prove that $d(f\omega) = df\wedge\omega + fd\omega$ for any k-form ω on M. [3]

6 (a) Define a Lie group. Let ω be a left invariant k-form on a Lie group G. Show that dω is a left invariant (k+1)-form. [2+2]

(b) Let G be a Lie group and $f: G \to G$ be a diffeomorphism such that $f(a) = a^{-1}, \forall\, a \in G$. Prove that ω is a left invariant form if and only if $f^*\omega$ is right invariant form. [6]