M.A./M.Sc. Semester III Examination, 2019 (Old pattern under CDOE)
Subject: Mathematics (Applied Stream)
Paper: MAG302

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

Group-A (Methods of Applied Mathematics-II)
(Marks: 27)

Answer any **three** questions. Only **first three** answers will be evaluated. 9×3 = 27

1. (a) Write down the four properties satisfied by Riemann Green's functions for a hyperbolic partial differential equation. [2]

   (b) Obtain the solution of $\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{x+y}$ such that $z = 0, p = \frac{2y}{x+y}$ on $y = x$ using the Riemann Green's function. [7]

2. (a) State and prove the maximum-minimum theorem for an elliptic equation. [5]

   (b) Can a Neumann problem be converted into a Dirichlet problem? Justify your answer. [4]

3. (a) Find the temperature in a laterally insulated bar of length a whose ends are insulated assuming that the initial temperature is *f(x)*=$\begin{cases} x \text{ if } 0 < x \leq a/2 \\ a - x \text{ if } \frac{a}{2} < x < a \end{cases}$. [5]

   (b) Find the Green's function for the equation $\frac{d^2 u}{dx^2} = f(x), 0 \leq x \leq 1$ subject to the boundary conditions u(0)-u'(0)=0 and u(1)+u'(1)=0 . [4]

4. (a) Illustrate an example on the fact that the solution of a wave equation should be continuously dependent on its initial conditions. [5]

   (b) State and find the solution of D'Álembert's problem for wave equation. [4]

5. (a) How will you find the type of a second order partial differential equation for more than two independent variables? [5]

   (b) Set an example of ill-posed partial differential equation. [4]

Group-B (Theory of Electro Magnetic Fields)
(Marks: 18)

Answer any **two** questions. Only **first two** answers will be evaluated. 9×2 = 18

1. Write down Biot-Savat law. Hence obtain the divergence and the curl of the magnetic field vector. [3+3+3]

2. Using Maxwell's equations prove that empty space supports the propagation of electromagnetic waves at a constant speed, 3.0×10^8 m/s. [9]

3. Define electromagnetic potentials. Hence explain the concept of gauging of those potentials. [9]

M.A./M.Sc. Semester III Examination, 2019 (Old pattern under CDOE)

Subject: Mathematics (Pure Stream)

Paper: MPG302

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Group-A (Graph Theory)

(Marks: 36)

Answer any **four** questions. Only **first four** answers will be evaluated. 9×4 = 36

1. (a) Let G be a connected graph. Show that every walk in G has a path. [5]

   (b) Prove that a connected graph is semi-Eulerian if and only if there exist exactly two vertices of odd degree. [4]

2. (a) Let G be a simple graph with n vertices and k components. Show that the maximum number of edges in G is $\frac{(n-k)(n-k+1)}{2}$. [5]

   (b) What do you mean by adjacency matrix of a digraph? Illustrate with an example. [2+2]

3. (a) State and prove 5-colour theorem. [1+4]

   (b) Define bipartite graph. Show that every bipartite graph is 2-chromatic. [2+2]

4. (a) Let G be a connected planar graph with e edges, n vertices and f faces. Prove that $n - e + f = 2$. [5]

   (b) Let $G(V,E)$ be a simple connected graph with n vertices such that $d(v) \geq \frac{n}{2}, \forall\, v \in V$. Prove that G is Hamiltonian. [4]

5. (a) When is a graph said to be minimally connected? Show that a graph G is a tree if and only if G is minimally connected. [1+4]

   (b) State Kuratowski Theorem. [2]

   (c) Give an example of two graphs that are isomorphic to each other. [2]

6. (a) State and prove Konig theorem. [5]

   (b) Prove that every vertex of an Eulerian graph is of even degree. [4]

Group-B (Set Theory-I)

(**Marks: 9**)

Answer any **one** question. Only **first one** answer will be evaluated. $9\times1 = 9$

1 (a) State Axiom of Choice and Zorn's lemma. Prove that Axiom of Choice implies Zorn's lemma. [5]

(b) When is a set said to be well ordered? State Well Ordering theorem. Show that Well Ordering theorem implies Axiom of Choice. [4]

2 Define the ordering $\leq$ on the class of cardinal numbers. State and prove Schroder-Bernstein theorem. [9]