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M.A./M.Sc. Semester III Examination, 2019 (Old pattern under CDOE)
Subject: Mathematics (Applied Stream)
Paper: MAG303

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Group-A (Continuum Mechanics-II)
(Marks: 27)

Answer any **three** questions. Only **first three** answers will be evaluated. 9×3 = 27

1. (a) Show that the principal directions of strain corresponding to the principal stresses are orthogonal to each other. [4]

   (b) Show that $\frac{DJ}{Dt} = J\upsilon^k_{,k}$ where J is the Jacobian of the transformation from material co-ordinates to spatial co-ordinates, υ is the velocity. [5]

2. (a) State Cauchy's fundamental theorem for stress. With reference to contact forces, state Euler's first law of motion for a continuous medium. From this law, obtain stress equation of motion due to Cauchy. [2+2+3]

   (b) Find the strain components for the following displacement field: [2]

   $u = -\alpha yz, \upsilon = \alpha zx, w = 0,$ where α is a constant.

3. (a) Under what conditions the following is a possible system of strain components of an elastic body? [5]

   $e_{xx} = a + b(x^2 + y^2) + x^4 + y^4, e_{yy} = \alpha + \beta(x^2 + y^2) + x^4 + y^4,$

   $e_{xy} = \gamma + \delta xy(x^2 + y^2 + c^2), e_{zz} = e_{yz} = e_{zx} = 0$ where $a, b, c, \alpha, \beta, \gamma, \delta$ all are constants.

   (b) Define deformation function and obtain deformation gradient tensor for the motion of a deformable body. [4]

4. (a) What do you mean by bounding surface? Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t - 1 = 0$ is a possible bounding surface for an incompressible fluid. [2+3]

   (b) Define Cauchy's stress quadric at a point of a continuous medium. [4]

5. (a) The state of stress at a point of a continuous medium is given by [5]

   $\left[\sigma_{ij}\right] = \begin{bmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{bmatrix}$ where a, b, c are constants and σ is some stress

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value. Determine the constants a, b, c so that the stress vector on a plane normal to $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ vanishes.

(b) State the fundamental boundary value problems of elastostatics. [4]

Group-B (Dynamical Systems)
(Marks: 18)

Answer any **two** questions. Only **first two** answers will be evaluated. 9×2 = 18

1. (a) With the help of flow concept discuss the local stability of the fixed points for the system $\dot{x} = (x^2 - 1)$. [4]
   (b) State Hartman-Grobman theorem and explain its significance. [5]
2. (a) Define a limit cycle. Find the limit cycle for the system $\dot{r} = r(1 - r^2), \dot{\theta} = 1$. [5]
   (b) Show that Lorenz system is dissipative. [4]
3. (a) Define Lyapunov function and state Lyapunov theorem for stability. [4]
   (b) Discuss pitchfork bifurcation for a one-dimensional system. [5]

M.A./M.Sc. Semester III Examination, 2019 (Old pattern under CDOE)
Subject: Mathematics (Pure Stream)
Paper: MPG303

Time: 2 Hours Full Marks: 45

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[Notation and symbols have their usual meaning]

Group-A (Set Theory-II & Mathematical Logic)
(Marks: 27)

Answer any **three** questions. Only **first three** answers will be evaluated. 9×3 = 27

1. (a) If α, β, γ are cardinal numbers, then show that $\alpha^\beta \alpha^\gamma = \alpha^{\beta+\gamma}$. [5]
   (b) If a and c denote the cardinal numbers of the set of natural numbers and the set of real numbers respectively, then show that $2^a = c$. [4]
2. (a) For any two ordinal numbers α and β, show that exactly one of the following holds: $\alpha < \beta,\ \alpha = \beta,\ \beta < \alpha$. [9]
3. (a) Give the definitions of ‘tautology’ and ‘contradiction’ with example. Give an example of a statement form which is neither a tautology nor a contradiction. [6]
   (b) If the statement forms $(A \to B)$ and A are both tautologies then examine if B is a tautology. [3]

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4. (a) Determine whether the following argument is correct. 'If the prices are high then the wages are high. Prices are high or there are price controls. If there are price controls then there is not an inflation. There is an inflation. Therefore the wages are high.' [4]

(a) Prove that every truth function corresponds to a statement form containing as connectives only $\wedge$ and $\sim$ or only $\vee$ and $\sim$ or only $\Rightarrow$ and $\sim$ [5]

5. (a) For any well formed formulas $\mathcal{A}$ and $\mathcal{B}$ of axiomatic theory L prove that [3]

$$\sim \mathcal{A} \Rightarrow (\mathcal{A} \Rightarrow \mathcal{B})$$

is a theorem of L.

(b) Symbolize the following statements: [6]

(i) Some people are good for all time and all people are good for sometime but not all people are good for all time.

(ii) All lawyers admire all judges but there are judges who do not admire all lawyers.

(iii) Some groups are abelian and some abelian groups are cyclic but not all groups are cyclic.

Group-B (Functional Analysis-II)

(**Marks: 18**)

Answer any **two** questions. Only **first two** answers will be evaluated. 9×2 = 18

1 (a) Define a completion of a metric space. If (X, d) is a discrete metric space, then find the completion of (X, d). [1+1]

(a) Let M be a compact subset of $C[a, b]$, where $C[a, b]$ is equipped with sup metric. Prove that M is uniformly bounded. [3]

(c) If a normed linear space X has the property that the closed unit ball $M = \{x \in X: \| x \| \leq 1\}$ is compact, then prove that X is finite dimensional. [4]

2 (a) Give an example with proper justification of an infinite series in some normed linear space which is absolutely convergent but not convergent. [4]

(b) Let $\{T_n\}_{n\in\mathbb{N}}$ be a sequence of bounded linear operators $T_n : X \to Y$ from a Banach space X into a normed linear space Y such that $\{ \| T_n(x) \| \}_{n\in\mathbb{N}}$ is bounded for every $x \in X$. Prove that the sequence $\{ \| T_n \| \}_{n\in\mathbb{N}}$ is bounded. [5]

3 (a) Let X and Y be two Banach spaces over the same field F and $T: X \to Y$ be a linear operator. Prove that T is continuous if and only if its graph is a closed subspace of $X \times Y$, where $X \times Y$ is equipped with the norm $\|(x, y)\| = \|x\| + \|y\|$, $\forall (x, y) \in X \times Y$. [5]

(b) Let X be a normed linear space and M be a proper closed linear subspace of X. Let ϵ be a real number with $0 < \epsilon < 1$. Prove that there exists $x_\epsilon \in X$ such that $\|x_\epsilon\| = 1$ and $dist\ (x_\epsilon, M) \geq 1 - \epsilon$. [4]