,

M.A./M.Sc. Semester III Examination, 2019 (Old pattern under CDOE)
Subject: Mathematics (Applied Stream)

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

Paper: MAS304 (Viscous Flows, Boundary Layer Theory and Magneto Hydrodynamics-I)

Answer any **five** questions. Only **first five** answers will be evaluated. 9×5 = 45

1. Write Stokes' hypothesis for continuous flow in fluid medium. Establish the relation $\sigma_{ij} = -p\delta_{ij} + 2\mu e_{ij} + \lambda e_{kk}\delta_{ij}$, $i, j = 1, 2, 3$ and λ, μ are constants. [3+6]
2. Determine the velocity distribution in the flow of an incompressible viscous fluid due to the motion of an infinite plate which oscillates harmonically with a given velocity $U\cos(nt)$. [9]
3. Obtain two-dimensional Prandtl boundary layer equations of an incompressible fluid for flow past a flat plate. [9]
4. Derive Karman's integral equation for steady two-dimensional boundary layer flow. [9]
5. Discuss the flow of viscous incompressible fluid in the neighborhood of a stagnation point in a plane. [9]
6. (a) Give the definition of flow separation. Write its criteria. [4]
   (b) Define Reynolds number (Re) for viscous fluid motion. Write its significances when the values of Re are small and large. [5]
7. (a) Define energy thickness parameter in a boundary layer flow. Give its physical significance. [4]
   (b) A plate of length l is placed in an uniform stream U in the direction of its length. Compute the boundary layer thickness for the velocity $u = U\sin(\pi\eta/2)$, where $\eta = y/\delta$. [5]

M.A./M.Sc. Semester III Examination, 2019 (Old pattern under CDOE)
Subject: Mathematics (Pure Stream)

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

,

Paper: MPS304 (Differential Geometry of Manifolds-I)

Answer any **five** questions. Only **first five** answers will be evaluated. $9\times5 = 45$

1. Define a smooth manifold. Show that the n-sphere S^n is an n-dimensional smooth manifold. [3+6]

2. (a) Define tangent space at a point of a smooth manifold and obtain a basis such a space. [2+4]

   (b) Determine the integral curve for the vector field $X = x^2 \frac{\partial}{\partial x^1} - x^1 \frac{\partial}{\partial x^2}$ in $\mathbb{R}^2$. [3]

3. (a) Let M be a smooth manifold of dimension n and $X, Y \in \chi(M)$. Define $[X,Y]$ and derive its local expression. Also show that $[X, Y] \in \chi(M)$. [1+3+3]

   (b) Is the map $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ a chart? Justify your answer. [2]

4. (a) Define one-parameter group of differentiable transformations. Can a tangent vector field X on a smooth manifold M determine a one-parameter group of transformations? Give reasons. [2+3]

   (b) Let $f: M \to N$ be a smooth map, where M and N are two smooth manifolds. For any$r$-form on N, show that $f^*(d\omega) = d(f^*\omega)$, where f_* and f^* are the pushforward and pullback map of f and d denotes the exterior differentiation. [4]

5. (a) Let M and N be two smooth manifolds with $dim\ M = m$ and $dim\ N = n$ and $f: M \to N$ be a smooth map. When f is said to be an imbedding?
   If (M, f) is a submanifold of N and M is compact then show that $f: M \to N$ is a regular imbedding. [1+3]

   (b) Prove that the map $\varphi: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $\varphi(x, y, z) = (e^{2y} + e^{2z}, e^{2x} - e^{2y}, x - y)$ is a C^∞-diffeomorphism. [5]

6. (a) Define principal fibre bundle, linear frame bundle and associate principal bundle. [6]

   (b) Let $X = xy\frac{\partial}{\partial x} + x^2\frac{\partial}{\partial z}$ be a vector field on $\mathbb{R}^3$ and $f: \mathbb{R}^3 \to \mathbb{R}$ be defined by $f(x, y, z) = x^2y$. Compute $(fX)_{(1,1,0)}$ and $(Xf)_{(1,1,0)}$. [3]

7. (a) Prove that Lie algebra of the Lie group G is isomorphic to T_eG, $e \in G$ is the identity of G. [5]

   (b) Deduce Maurer-Cartan structure equation of a Lie group G. [4]

Paper: MPS304 (Advanced Functional Analysis-I)

Answer any **five** questions. Only **first five** answers will be evaluated. $9\times5 = 45$

1. Define a topological vector space. If X is a topological vector space then prove the following: [2+2+2+3]

   (i) If Y is a subspace of X then so is $\overline{Y}$.

   (ii) If E is a bounded subset of X then so is $\overline{E}$.

   (iii) If C is a convex subset of X, then interior of C and closure of C are also convex sets in X.

,

2. (a) Let X be a topological vector space. For $a \in X$, prove that the operator $T_a : X \to X$ defined by $T_a(x) = a + x$ is a homeomorphism. Hence prove that sum of two open sets in X is also an open set in X. [2+2]

   (b) When is a topological vector space said to be locally compact? Prove that a locally compact topological vector space is of finite dimension. [1+4]

3. Let X be a topological vector space and K and C are subsets of X such that K is compact and C is closed satisfying $K \cap C = \phi$. Show that there exists a neighborhood V of θ, the zero vector of X such that $(K+V) \cap (C+V) = \phi$. Hence show that every topological vector space is Hausdorff. [6+3]

4. When is a Hausdorff topological vector space said to be normable? Prove that a topological vector space X is normable if and only if zero vector of X has a convex bounded neighbourhood in X. [2+7]

5. (a) Define a strictly convex normed linear space. Give an example with justifications of a normed linear space which is not strictly convex. [2+2]

   (b) Show that the space $l_p \, (1 < p < \infty)$ is uniformly convex. [5]

6. (a) When is a topological vector space said to be locally convex? Show that in a locally convex topological vector space, the balanced closed convex neighbourhoods of θ form a fundamental system of neighbourhoods of θ. [2+5]

   (b) Show that every topological vector space is connected. [2]

7. (Let f be a non-zero linear functional on a topological vector space X. Prove that the following statements are equivalent. [9]

   (i) f is continuous,

   (ii) the null space $N(f)$ is closed,

   (iii) $N(f)$ is not dense in X and

   (iv) f is bounded in some neighbourhood of θ.