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**M.A./M.Sc. Semester III Examination, 2019 (Old pattern under CDOE)**
**Subject: Mathematics (Applied Stream)**

Time: 2 Hours Full Marks: 45

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*[Notation and symbols have their usual meaning]*

**Paper: MAS305 (Advanced Operations Research-I)**

Answer any **five** questions. Only **first five** answers will be evaluated. 9×5 = 45

1. (a) State and prove the sufficient condition for optimality of Lagrange multiplier method for solving constrained optimization problem with equality constraints [4]

   (b) Solve the following problem by Lagrange multiplier method [5]

   Optimize $z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$

   subject to $x_1 + x_2 + x_3 = 7$

2. (a) Show that the Kuhn Tucker necessary conditions for the optimization problem [4]

   Minimize $f(x)$

   subject to $g_i(x) \le 0, \quad i = 1, 2, \cdots, m$

   and $h_j(x) = 0, \quad j = 1, 2, \cdots, l$

   are also sufficient conditions if $f(x)$ is convex and $g_i(x)$, $i = 1, 2, \cdots, m$ are convex functions of $x$ and $h_j(x)$ are linear.

   (b) Use the Kuhn-Tucker necessary conditions to solve the following optimization problem: [5]

   Maximize $z = 2x_1 - x_1^2 + x_2$

   subject to $2x_1 + 3x_2 \le 6$, $2x_1 + x_2 \le 4$ and $3x_1 + 9x_2 = 16$

3. (a) If the iterative sequence $\left\{x^{(k)}\right\}$ be defined as [4]

   $$x^{(k+1)} = x^{(k)} + \lambda^{(k)} d^{(k)}$$

   where $d^{(k)}$ is given by

   $$d^{(k)} = -M_k \nabla f\left(x^{(k)}\right)$$

   and also, if $\nabla f\left(x^{(k)}\right) \ne 0$ and $M_k$ is positive definite, then show that the iterative procedure possesses decent property.

   (b) Using steepest descent method, minimize $f(x) = x_1^2 + x_2^2 + 2gx_1 + 2fx_2 + c$ starting from the point $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$. [5]

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4. (a) Write down Wolfe's algorithm for solving quadratic programming problem. [4]

   (b) Use Beale's method for solving the quadratic programming problem: [5]

   Maximize $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

   subject to $x_1 + 2x_2 \le 2$ and $x_1, x_2 \ge 0$

5. (a) Discuss Fibonacci search method for solving one-dimensional non-linear minimization problem. [4]

   (b) Using Fibonacci search method [5]

   Maximize $f(x) = \begin{cases} \dfrac{2x+3}{6} & \text{for } x \le 3 \\ -x+6 & \text{for } x > 3 \end{cases}$

   in the interval $[-1, 5]$ (consider 6 experimental points).

6. (a) Define Slater's, Karlin's and strict constraint qualifications. [3]

   (b) Show that [3+3]

   (i) Slater's constraint qualification and Karlin's constraint qualification are equivalent.

   (ii) The Strict constraint qualification implies Slater's constraint qualification and Kerlin's constraint qualification.

7. (a) State and prove Max-flow Min-cut theorem for a network flow problem. [4]

   (b) Find the maximum flow in the graph with the following arcs and arc capacities, flow in each arc being non-negative. Arc $(v_j, v_k)$ is denoted as (*j*, *k*). $v_a$ is the source and $v_b$, the sink. [5]

| Arc | (*a*, 1) | (*a*, 2) | (*a*, 3) | (1, 4) | (1, 5) | (1, 6) | (2, 4) | (2, 5) |
|---|---|---|---|---|---|---|---|---|
| Capacity | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 |

| Arc | (2, 6) | (3, 4) | (3, 5) | (3, 6) | (4, *b*) | (5, *b*) | (6, *b*) |
|---|---|---|---|---|---|---|---|
| Capacity | 1 | 1 | 1 | 1 | 2 | 2 | 2 |

**M.A./M.Sc. Semester III Examination, 2019 (Old pattern under CDOE)**

**Subject: Mathematics (Pure Stream)**

Time: 2 Hours Full Marks: 45

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**Paper: MPS305 (Operator Theory and Applications I)**

Answer any **five** questions. Only **first five** answers will be evaluated. 9×5 = 45

1. (a) Let $X$ be a normed linear space with dual $X^{/}$. Define annihilator of a subset of $X$ and $X^{/}$. Show that the annihilator of a subset and the orthogonal complement of that subset coincide in a Hilbert space. [5]

   (b) Let $T_1: X \to X$ and $T_2: X \to X$ be two bounded linear transformations and $T_1{}^{/}$ and $T_2^{/}$ be its conjugates respectively where $X$ is a normed linear space. Show that (i) $(T_1+T_2)^{/} = T_1^{/} + T_2{}^{/}$ and (ii) $(T_1T_2)^{/} = T_2^{/}T_1{}^{/}$. [4]

2. (a) Show that a surjective symmetric operator T: $D_T \to X$ is self adjoint where $D_T$ is a dense subspace of a complex Hilbert space $X$. [3]

   (b) Let $T \in B(\mathrm{X}, \mathrm{Y})$ where $X$ and $Y$ are complex Hilbert spaces. Show that (i) $(\overline{R(T)})^{\perp} = N(T^*)$(ii) $\overline{(R(T^*)})^{\perp} = N(T)$(iii)$\|T^*T\| = \|T\|^2$ [6]

3. (a) Let $E_1$ and $E_2$ be two orthogonal projections on the closed subspaces $M_1$ and $M_2$ of a complex Hilbert space $X$ respectively. Show that the following conditions are equivalent. (i)$E_1 \leq E_2$(ii) $\|E_1x\| \leq \|E_2x\|$ for all $x \in X$ (iii) $M_1 \subset M_2$ (iv) $E_2E_1 = E_1$ (v) $E_1E_2 = E_1$. [4]

   (b) Let $E_1$ and $E_2$ be two orthogonal projections on the closed subspaces $M_1$ and $M_2$ of a complex Hilbert space $X$ respectively. Prove that $E_1 + E_2$ is an orthogonal projection on $\overline{[M_1 \cup M_2]} = M_1 + M_2$ if and only if $E_1$ is orthogonal to $E_2$. [5]

4. Let $X$ be a complex Hilbertspace and let $A, B \in B(X, X)$ and$AB$=$BA$ . Show that $A \geq 0$,$B \geq 0$ implies $AB \geq 0$. [9]

5. (a) If $A$ and $B$ are two compact operators defined on a normed linear space $X$ into a normed linear space $Y$, then show that $A+ B$ is compact and $\alpha A$ is compact where $\alpha$ is a scalar. [4]

   (b) Let $H_1$ and $H_2$ be two complex Hilbert spaces and let $T: H_1 \to H_2$ be a compact linear operator. Prove that the adjoint operator $T^*$ is compact. [5]

6. (a) Prove that the uniform limit of a sequence $\{A_n\}$ of compact operators defined on a normed linear space $X$ into a normed linear space $Y$ is compact. [6]

   (b) Let $H_1$ and $H_2$ be two normed linear spaces and let $T: H_1 \to H_2$ be a compact linear operator. Show that range of $T$ is separable. [3]

7. (a) Let $X$ be a complex normed linear space and let $A: X \to X$ be a compact linear operator and $\lambda \neq 0$. If $y = \lambda x - Ax$ has a solution for arbitrary $y \in X$ , then prove that the equation $\lambda x - Ax = 0$ has only trivial solution $x = 0$. [6]

   (b) Let $X$ be a complex normed linear space and let $A: X \to X$ be a compact linear operator and $\lambda \neq 0$. Then show that $y = \lambda x - Ax$ is solvable for those and only those $y$ belonging to $a_{N(\lambda I - A^{/})}$. [3]