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M.A./M.Sc. Semester I Examination, 2019 (Old pattern under CDOE)

Subject: Mathematics

Paper: MCG102

Time: 2 Hour Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning.]

Write the answer to Questions of each Group in separate books.

Group–A (Linear Algebra)

[Marks: 27]

Answer any **three** questions. Only **first three** answers will be evaluated. $9\times 3 = 27$

1. (a) Let $T \in L(\mathbb{R}^2, \mathbb{R}^3)$ be defined by $T(a,b) = (a+3b, 0, 2a-4b), a, b \in \mathbb{R}$. Find the matrix of T with respect to the standard ordered basis of $\mathbb{R}^2$ and $\mathbb{R}^3$. [3]

   (b) Use Gram-Schmidt process to obtain an orthonormal basis of the subspace of the Euclidean space $\mathbb{R}^4$ with standard inner product generated by the linearly independent set {(1,1,0,1), (1,1,0,0), (0,1,0,1)}. [6]

2. (a) Diagonalize the symmetric matrix $A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$. [5]

   (b) Let $T: P_2(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ be a linear transformation defined by [4]

$$T\big(f(x)\big) = \begin{pmatrix} f(1) - f(2) & 0 \\ 0 & f(0) \end{pmatrix}.$$

   Find a basis for $R(T)$ and hence show that rank of T is 2. [$P_2(\mathbb{R})$ stands for the space of all polynomials over $\mathbb{R}$ of degree ≤ 2].

3. (a) Let V and W be vector spaces over the same field F and V is of finite dimensional. If $T \in L(V, W)$ then show that nullity of T + rank of T = dim V. [7]

   (b) State Sylvester's law of inertia. [2]

4. (a) What do you mean by a minimal polynomial of a square matrix? Show that minimal polynomial of a square matrix is unique. [1+4]

   (b) Find the minimal polynomial of the matrix $A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}$. [4]

5. (a) When are two matrices said to be similar? Show that two similar matrices have the same eigenvalue. [2+3]

   (b) Reduce the quadratic form $5x^2 + y^2 + 10z^2 - 4yz - 10zx$ to the normal form and show that it is positive definite. [4]

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Group–B (Modern Algebra)

[Marks: 18]

Answer any **two** questions. Only **first two** answers will be evaluated. $9 \times 2 = 18$

1. (a) Let G be a group and $Z(G)$ be its center. If σ is an automorphism of G, show that $Z(G) \subseteq \sigma(Z(G))$. [5]

   (b) Let G be a group and N be a maximal normal subgroup of G. Show that $[G:N]$ can never be 6. [4]

2. (a) Let $F[x]$ be a commutative polynomial ring over a ring F with 1. Show that non-unit elements of $F[x]$ is precisely the non-constant polynomials over F if and only if F is a field. [3+4]

   (b) Give an example to show that the result in 2(a) is not true if F is not a field. [2]

3. (a) Show that in a finite commutative ring with 1, every prime ideal is maximal. Give an example with justification of a commutative ring where a maximal ideal is not prime. [3+3]

   (b) Show that in PID, g.c.d of two elements always exists. [3]