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M.A./M.Sc. Semester I Examination, 2019 (Old pattern under CDOE)

Subject: Mathematics

Paper: MCG104

Time: 2 Hour Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning.]

Write the answer to Questions of each Group in separate books.

Group–A (Ordinary Differential Equations & Special Functions)

[Marks: 27]

Answer any **three** questions. Only **first three** answers will be evaluated. 9× 3 = 27

1. (a) For the Legendre polynomial of degree *n* establish the recurrence relation, [4]

$$(n+1)P_{n+1}(z)=(2n+1)zP_n(z)-nP_{n-1}(z)$$

(b) Obtain the power series solution of the Laguerre equation, [5]

$z\frac{d^2w}{dz^2}+(1-z)\frac{dw}{dz}+\lambda w=0$ in the neighborhood of $z=0,\lambda$ being a parameter.

2. (a) Examine singularity at the points $z=0,1$ for the equation, [3]

$$(1-z)\frac{d^2w}{dz^2}+z\frac{dw}{dz}-w=0.$$

(b) Find two linearly independent solutions of the equation $3z\frac{d^2w}{dz^2}+\frac{dw}{dz}-2w=0$ about $z=0$. [6]

3. (a) Show that for Legendre polynomial, $P_n(-z)=(-1)^nP_n(z)$ and hence deduce that $P_n(-1)=(-1)^n$. [6]

(b) Find e^A for $A=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. [3]

4. (a) Find the general solution of the linear system, $\dot{x}=10x-y,\ \ \dot{y}=25x+2y$. [6]

(b) Determine the nature of the critical point for the system $\dot{x}=-x-2y,\ \dot{y}=4x-5y$. [3]

5. (a) Show that $\frac{dy}{dx}=3y^{2/3}$ with $y(0)=0$ has solutions but not unique. [3]

(b) Obtain three consecutive equilibrium points and their types of stability for the non-linear simple pendulum $\dot{x}=y,\dot{y}=-\sin(x)$. [6]

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Group–B (Operations Research-I)
[Marks: 18]

Answer any **two** questions. Only **first two** answers will be evaluated. 9× 2 = 18

1. (a) What is the advantage of revised simplex method? Write down the computational procedure of standard form I of revised simplex method. [4]

   (b) Using revised simplex method, solve the following L.P.P.: [5]

   $$\text{Maximize } z = 6x_1 - 2x_2 + 3x_3$$
   $$\text{subject to } 2x_1 - x_2 + 2x_3 \le 2$$
   $$x_1 + 4x_3 \le 4, x_1, x_2, x_3 \ge 0.$$

2. (a) Write down the computational procedure of bounded variable technique. [3]

   (b) Using bounded variable technique, solve the following L.P.P.: [6]

   $$\text{Maximize } z = 3x_1 + 5x_2 + 2x_3$$
   $$\text{subject to } x_1 + 2x_2 + 2x_3 \le 14$$
   $$2x_1 + 4x_2 + 3x_3 \le 23, 0 \le x_1 \le 4, 2 \le x_2 \le 5, 0 \le x_3 \le 3.$$

3. (a) What is the difference between Assignment problem and Transportation problem? Write down the solution procedure of Hungarian method for solving Assignment problem. [4]

   (b) Four different jobs can be done on four different machines. The following matrix gives the cost in rupees of producing job J_i on machine M_j. [5]

	M_1	M_2	M_3	M_4
J_1	15	35	105	75
J_2	75	27	10	25
J_3	30	20	12	55
J_4	67	65	45	40

   How should the jobs be assigned to the various machines so that the total cost is minimized?