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M.A./M.Sc. Semester I Examination, 2019 (Old pattern under CDOE)

Subject: Mathematics

Paper: MCG105

Time: 2 Hour Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning.]

Write the answer to Questions of each Group in separate books.

Group–A (Principle of Mechanics-I)

[Marks: 27]

Answer any **three** questions. Only **first three** answers will be evaluated. $9\times 3 = 27$

1. (a) For a scleronomous dynamical system, show that the kinetic energy of a particle is a homogeneous quadratic function of the generalized velocities. [5]

   (b) The Lagrangian, L, for the motion of a particle of unit mass is given by , [4]

   $L=\frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)-V+A\dot{x}+B\dot{y}+C\dot{z}$ where V, A, B, C all are functions of x, y, z.

   Show that the equations of motion are $\ddot{x}=-\frac{\partial V}{\partial x}+\dot{y}\left(\frac{\partial B}{\partial x}-\frac{\partial A}{\partial y}\right)-\dot{z}\left(\frac{\partial A}{\partial z}-\frac{\partial C}{\partial x}\right)$ and similar equations.

2. (a) Assuming Lagrange's equations of motion in the form, $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right)-\frac{\partial T}{\partial q_k}=Q_k^{/}$ $(k=1,2,...,n)$ (where T is the kinetic energy and $Q_k^{/}$'s are the generalized forces) obtain Rayleigh's dissipation function and hence deduce Lagrange's equations of motion for dissipative forces. [5]

   (b) Find the Routhian function for the motion of a particle of unit mass moving in a central force field, where the force varies inversely as the square of the distance from the centre of force. [4]

3. (a) Establish the Euler-Lagrange differential equation to determine the function $y=y(x)$ for which the integral $\int_a^c f\left(x,y,y'\right)dx$ remains stationary, where the functions $y(x)$ and f satisfy the requisite conditions. [Here, $y'=\frac{dy}{dx}$]. [5]

   (b) Show that the shortest distance between any two points in a plane is a straight line. [4]

4. (a) Derive the Hamilton's principle from Lagrange's equations of motion. [5]

   (b) Obtain Hamilton's equations of motion of a dynamical system whose Hamiltonian is given by $H(q_1,p_1,q_2,p_2)=q_1p_1-q_2p_2-aq_1^2+bq_2^2$. Hence solve the problem. [4]

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5. (a) Prove that the Poisson bracket of two dynamical variables is invariant under canonical transformation. [5]

(b) Show that the transformation, $(P,Q) \to (p,q)$, defined by, $Q = -p, P = q + \lambda p^2$ is a canonical transformation. [4]

Group–B (Numerical Analysis)

[Marks: 18]

Answer any **two** questions. Only **first two** answers will be evaluated. 9× 2 = 18

1. Taking $p = 1, q = 2$ as initial approximations, apply Bairstow's method to extract a quadratic factor $x^2 + px + q$ from following polynomial equation $x^4 - 15x^2 - 10x + 24 = 0$ by performing three iterations. [9]

2. (a) Derive Adams-Bashforth method for numerical solution of $\frac{dy}{dx} = f(x,y)$ with $y(x_0) = y_0$. [5]

(b) Using the two point Gaussian quadrature rule, evaluate the integral, [4]

$$I = \int_0^1 \frac{1}{1+x}dx$$

3. (a) Using Scmidt's method, find the numerical solution of [5]

$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}, 0 < x < 0.5, 0 < t < 0.02,$

subject to

$u(x,0) = x^2(0.5 - x), 0 \le x \le 0.5$ and $u(0,t) = u(0.5,t) = 0, t \ge 0$, taking step lengths as 0.1 and 0.01 in the x and t directions respectively.

(b) Show that the Crank-Nicolson numerical scheme for solving one-dimensional heat conduction equation is unconditionally stable. [4]