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M.A./M.Sc. Semester IV Examination, 2021 (CBCS)

Subject: Mathematics (Applied Stream)

Course: MMATA401 (Fluid Mechanics)

Time: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **four** questions. Only **first four** answers will be evaluated. $10\times4 = 40$

1. (a) State and prove Kelvin's circulation theorem for the motion of an incompressible inviscid fluid in a simply connected region. [5]

   (b) If $udx+vdy+wdz = d\theta+\lambda d\chi$, where u,v,w are the velocity components and θ,λ,χ are functions of x,y,z,t. Prove that the vortex lines at any time are the lines of intersection of the surfaces λ = constant and χ = constant. [5]

2. (a) If a region lying wholly in a liquid be bounded by a spherical surface and if the liquid be moving irrotationally, then show that the mean value of the velocity potential over the surface is independent of the radius of the surface and is equal to its value at the centre of the sphere. [4]

   (b) For an irrotational motion in a doubly connected region, show that the circulation does not vanish and has a constant value along a closed circuit. [3]

   (c) Explain the physical significance of stream function for two dimensional motion of an incompressible inviscid fluid. [3]

3. (a) State and prove Kutta-Joukowski's theorem. [5]

   (b) A sphere is at rest in an infinite mass of homogeneous liquid of density ρ, the pressure at infinity being π; show that if the radius R of the sphere varies in any manner, the pressure at the surface of the sphere in any time is [5]

$$\pi+\frac{1}{2}\rho\left[\frac{d^2}{dt^2}\left(R^2\right)+\left(\frac{dR}{dt}\right)^2\right]$$

4. (a) Derive Navier-Stokes' equations of motion of a viscous incompressible fluid in vector form. [6]

   (b) Define Reynolds number and interpret it physically. [2+2]

5. An incompressible viscous fluid flows along an elliptic pipe under uniform axial pressure gradient. Find the velocity distribution and rate of mass flux through it. Show that a circular pipe discharges at a greater rate than an elliptic one having the same cross sectional area. [4+3+3]

6. (a) Prove that in a circulation preserving motion, vortex lines move with the fluid. [4]

   (b) What do you mean by axi-symmetric motion? Obtain the equation satisfied by Stokes'-Stream function in cylindrical polar coordinates for such motion. [2+4]

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M.A./M.Sc. Semester IV Examination, 2021 (CBCS)

Subject: Mathematics (Pure Stream)

Course: MMATP401 & MMATP402

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Write the answers to questions of each course in separate books.

Course: MMATP 401 (Abstract Algebra III)

[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. 2×10 = 20

1. (a) Find the degree of the extension $\mathbb{Q}\left(\left(\sqrt{2}+\sqrt{3}\right)/\mathbb{Q}\right)$. [3]

   (b) Let D be an integral domain and F be a field in D such that $[D: F] < \infty$. Prove that D is a field. [4]

   (c) Show that for any field F, there exists a simple extension of F which is not algebraic over F. [3]

2. (a) Let F be a finite field with 81 elements. Construct an extension E of F such that $aut(E/F)$ has exactly 2021 elements. [5]

   (b) Find all automorphisms of $\mathbb{Q}(\sqrt[3]{3}, \sqrt{7})$. [5]

3. (a) Determine the Galois group of $(x^2-2)(x^2-3)(x^2-5)$. Determine all the subfields of the splitting field of this polynomial. [3+3]

   (b) Let F be the splitting field of $x^{2021}-2$ over $\mathbb{Q}$. Show that there exists a subfield E of F with $[F: E] = 43$. Further show that F is the Galois extension of E. [2+2]

Course: MMATP402 (Calculus of $\mathbf{R}^n$ –II)

[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. 2×10 = 20

1. (a) Let Q be a closed rectangle in $\mathbb{R}^n$ and $V(Q) \neq 0$. Show that Q is not a set of measure zero, where $V(Q)$ denotes volume of Q. [3]

   (b) Let Q be a rectangle in $\mathbb{R}^n$ and $f: Q \to \mathbb{R}$ be a bounded function. Assume that f is integrable over Q. If f is non-negative and $\int_Q f = 0$, then show that f vanishes on Q except for a set of measure zero. [4]

   (c) Let A, B be open sets in $\mathbb{R}^n$ such that $B \subset A$ and $f: A \to \mathbb{R}$ be a non negative, continuous function on A. If f is integrable over A, then show that f is integrable over B and also show that $\int_B f \leq \int_A f$. [3]

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2. (a) Let S be a bounded set in $\mathbb{R}^n$ and $f: S \to \mathbb{R}$ be a bounded continuous function. If f is integrable over S, then prove that f is integrable over $Int\ S$ and $\int_S f = \int_{Int\ S} f$, where $Int\ S$ denotes the set of all interior points of S. [5]

(b) Give an example with proper justification of a bounded set in $\mathbb{R}$ which is not rectifiable. [5]

3. (a) Let S be a bounded set in $\mathbb{R}^n$ and $f: S \to \mathbb{R}$ be a bounded function. If $S = S_1 \cup S_2$ and f is integrable over S_1 and S_2, then show that f is integrable over S and $S_1 \cap S_2$. Further show that $\int_S f = \int_{S_1} f + \int_{S_2} f - \int_{S_1 \cap S_2} f$. [4+1]

(b) Let W be an open set in $\mathbb{R}^2$ defined by $W = \{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 < 25 \}$. Using change of variables, evaluate $\int_W x^2 y^2$. [5]