# M.A./M.Sc. Semester IV Examination, 2021 (CBCS)
## Subject: Mathematics (Applied Stream)
## Course: MMATA402 & MMATA403

Time: 2 Hours                                                                                Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Write the answer to Questions of each Course in separate books.

### MMATA402 (Wavelet Analysis)

[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. $10\times2 = 20$

1. (a) Write down the Haar function. Plot it explaining the physical significances. [2+2]

   (b) Find the Fourier Transformation of the Haar function. Plot the transformed function. Interpret it. [4+2]

2. (a) If $\psi$ is a wavelet and $\phi$ is a bounded integrable function, then prove that the convolution function $\psi * \phi$ is a wavelet. [6]

   (b) If *U(a,b)* is a wavelet operator prove that *U(a,b)U(c,d)=U(ac, b+ad).* [4]

3 Show that the Legendre polynomials form an orthogonal system in the space to be stated by you. [10]

### MMATA403 (Dynamical Systems)

[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. $2\times 10 = 20$

1. Define orbital derivative. State Lyapunov theorem and apply it to show that origin is an asymptotically stable equilibrium point of the system: [2+2+6]

   $\dot{x} = -2y + yz - x^3, \dot{y} = x - xz - y^3, \dot{z} = xy - z^3$

2. (a) Write short note on saddle node bifurcation. [5]

   (b) Describe the bifurcation of the system $\dot{x} = x^3 - 5x^2 - (\mu - 8)x + (\mu - 4)$ [5]

3. (a) Find the fixed points of the one-dimensional map $f(x) = x + \sin x, x \in \mathbb{R}$. Also determine the basins of attraction. [1+2]

   (b) Show that $\{-1,1\}$ is an attracting 2-cycle of the map $f(x) = -x^{1/3}, x \in \mathbb{R}$. [3]

   (c) Prove that each solution of the equation $\frac{dx}{dt} + x = 0$ is asymptotically stable. [4]

**M.A./M.Sc. Semester IV Examination, 2021 (CBCS)**
**Subject: Mathematics (Pure Stream)**
**Course: MMATP403 (Topology-III)**

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

Answer any **four** questions. Only **first four** answers will be evaluated. 4× 10 = 40

1. (a) Let $\{S_n, n \in D, \geq\}$ be a net in a topological space $(X,\tau)$ and for each $n \in D$, $A_n = \{S_p : p \in D \text{ with } p \geq n\}$. Prove that a point $x_0 \in X$ is a cluster point of $\{S_n, n \in D, \geq\}$ if and only if $x_0 \in \overline{A_n}$ for every $n \in D$. [5]
   (b) Let $\{S_n, n \in D, \geq\}$ be a net in a topological space $(X,\tau)$. Prove that there exists a filter $\mathcal{F}$ on $X$ such that if $x \in X$ is a cluster point of the net $\{S_n, n \in D, \geq\}$, then $x$ is also a cluster point of the filter $\mathcal{F}$ and vice-versa. [5]
2. (a) Prove that a filter $\mathcal{F}$ on a set $X$ $(\neq \emptyset)$ is an ultrafilter if and only if any subset $A$ of $X$, which intersects every member of $\mathcal{F}$, belongs to $\mathcal{F}$. [5]

   (b) Let $\tau$ be the usual topology on $\mathbb{R}$, $A = \{a, b, c\}$ and $p: \mathbb{R} \to A$ be a function defined by $p(x) = \begin{cases} a, & if\ x > 0 \\ b, & if\ x < 0 \\ c, & if\ x = 0. \end{cases}$ [2]

   Find the quotient topology on $A$ generated by $p$.
   (c) Prove that every closed, surjective, continuous map from a topological space $(X,\tau)$ to a topological space $(Y,\sigma)$ is a quotient map. [3]
3. (a) Prove that the topological product of any collection of compact spaces is compact. [6]
   (b) Prove that every uniform space is regular. [4]
4. State and prove Embedding lemma and hence prove that a topological space $(X,\tau)$ is a Tychonoff space if and only if it is homeomorphic to a subspace of a cube. [5+5]
5. If $X$ is a Tychonoff space and $f$ is a continuous function from $X$ to a compact Hausdorff space $Y$, then prove that there is a continuous extension of $f$ which carries the Stone-Cech compactification $\beta(X)$ into $Y$. [10]
6. (a) Consider the Euclidean space $\mathbb{R}^n$ and $let\ x_0 \in \mathbb{R}^n$. Show that $\pi_1(\mathbb{R}^n, x_0) = \{0\}$, where 0 is the identity element of the group $\pi_1(\mathbb{R}^n, x_0)$. [5]
   (b) Show that the map $p: \mathbb{R} \to S^1$ given by the equation $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map, where $S^1$ is the unit circle in $\mathbb{R}^2$ endowed with the topology induced by the usual topology of the plane $\mathbb{R}^2$. [5]