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M.A./M.Sc. Semester IV Examination, 2021 (CBCS)

Subject: Mathematics (Applied Stream)

Course: MMATA404 & MMATG405

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Write the answer to Questions of each Course in separate books.

MMATA404 (Introduction to Quantum Mechanics)

[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2 = 20

1. Show that in the scattering of electromagnetic radiation from a stationary electron the change in wave length of the radiation depends only on the angle of scattering. [2+8]
2. Show that solution of the Schrodinger equation corresponding to the one-dimensional harmonic oscillator reduces to the solution of Hermite differential equation. [10]
3. If A and B be two Hermitian operators acting on the one-particle state space with [A, B] = $i\hbar I$, prove that $\Delta A \Delta B \geq \hbar / 2$. [10]

MMATG405 (Chaos and Fractals)

[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2 = 20

1. (a) Prove that conjugacy is an equivalence relation. [5]
   (b) Show that the logistic map $f_4(x)$ and the tent map are conjugate. [5]
2. (a) Explain topological mixing and Sensitive Dependence on Initial Conditions (SDIC) properties for a map $f : X \to X$. [6]
   (b) Find the Lyapunov exponent for the tent map. [4]
3. (a) White a short note on ‘Feigenbaum number’. [4]
   (b) Show that Cantor set is a self-similar fractal. Calculate the box dimension of von Koch curve. [6]

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M.A./M.Sc. Semester IV Examination, 2021 (CBCS)

Subject: Mathematics (Pure Stream)

Course: MMATP404 & MMATG405

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Write the answers to questions of each course in separate books.

Course: MMATP404 (Set Theory and Mathematical Logic)

[Marks 20]

Answer any **two questions**. Only **first two** answers will be evaluated. 2×10 = 20

1. (a) If A is any set, show that $card(A) < card(P(A))$, where $P(A)$ denotes the power set of A. [5]

   (b) Give an example with justifications of a valid argument which contains false premises and false conclusion. [2]

   (c) If P is a tautology containing statement letters $p_1, p_2, ..., p_n$ and Qarises from$P$by substituting forms $P_1, P_2, ..., P_n$respectively, then prove that $Q$is a tautology. Give an example in support of it. [2+1]

2. (a) If α, β and γ are cardinal numbers then show that $(\alpha\beta)^\gamma = \alpha^\gamma \beta^\gamma$. [5]

   (b) For any well-formed formula A, B, show that$\sim A \Rightarrow (A \Rightarrow B)$ is a theorem of axiomatic set theory$L$. [3]

   (c) For the given statement letters p and q, examine if the statement form $(p \Rightarrow q) \Leftrightarrow \big((\sim p) \vee q\big)$is a tautology. [2]

3. (a) Let $(A, \leq)$ be a well-ordered set. Then prove that for each $x \in A$, $A_x \cup \{x\}$ is either an initial segment of A or the whole of A. Show by an example that addition of ordinal numbers is not commutive. [3+2]

   (b) For the given statements A, B, C, D and E, prove that the following argument is valid [3]

   (i) $A \vee \big(B \Rightarrow D\big)$

   (ii) $\sim C \Rightarrow (D \Rightarrow E)$

   (iii) $A \Rightarrow C$

   (iv) $\sim C$

   $\therefore B \Rightarrow E$

   (c) Suppose that the statement letters p, q, r and s are assigned the truth values T, F, F and T respectively. Find the truth value of the following: [2]

   $(p \vee (\sim q) \vee r) \Rightarrow \big(s \vee (\sim s)\big)$, where 'T ' and 'F ' represent truth values 'True' and 'False' respectively for a given statement.

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Course: MMATP402 (Graph Theory)

[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. 2×10 = 20

1. (a) Show that the sum of the degrees of the vertices of a graph *G* is twice the number of edges in *G*. [5]

   (b) What do you mean by a graphical sequence? [1+4]
   Examine whether the sequence {3, 5, 3, 3, 3, 2, 2, 2, 1, 1, 1, 0, 0} is graphical or not.

2. (a) Prove that a connected graph *G* is a tree if and only if any two vertices of *G* are connected by a unique path. [5]

   (b) Show that every vertex of an Eulerian graph is of even degree. [5]

3. (a) Let $p_n(\lambda)$ be a chromatic polynomial of a graph *G* with *n* vertices. Prove that *G* is complete if and only if $p_n(\lambda) = \lambda(\lambda-1)(\lambda-2)\ldots(\lambda-n+1)$. [4]

   (b) Prove that a complete graph with five vertices has no dual. [4]

   (c) Draw a regular graph with six vertices. [2]