,

**M.A./M.Sc. Semester IV Examination, 2021 (CBCS)**

**Subject: Mathematics (Applied/Pure Stream)**

**Course: MMATAME407-1 (Advanced Optimization-II)**

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **four** questions. Only **first four** answers will be evaluated. $4\times 10 = 40$

1. (a) In a serial double-stage maximization problem if [5]

(i) the objective function $\psi_2$ is a separable function of stage returns $f_1(s_1, d_1)$ and $f_2(s_2, d_2)$

(ii) $\psi_2$ is a monotonic non-decreasing function of $f_1$ for every feasible value of $f_2$.

Prove that the problem is decomposable.

(b) Using Dynamic Programming, show that [5]

$$z = \sum_{i=1}^{n} p_i \log p_i$$

subject to the constraints

$$\sum_{i=1}^{n} p_i = 1 \quad \text{and } p_i > 0 \text{ for all } i,$$

is minimum when $p_1 = p_2 = \cdots = p_n = \dfrac{1}{n}$.

2. (a) A man is engaged in buying and selling identical items. He operates from a warehouse that can hold 500 items. In each month, he can sell any quantity that he chooses up to the stock at the beginning of the month. In each month, he can buy as much as the wishes for delivery at the end of the month so long as his stock does not exceed 500 items. For the next four months, he has the following forecasts of purchase costs and selling prices. [5]

| | | | | | |
|---|---|---|---|---|---|
| Month | : | 1 | 2 | 3 | 4 |
| Purchase cost | : | 27 | 24 | 26 | 28 |
| Selling price | : | 28 | 25 | 25 | 27 |

If he has a current stock of 200 units, what quantities should he sell and buy in the next four months? Find the solution using Dynamic Programming.

(b) Solve the reliability optimization problem with the following data: [5]

| $x_i$ | $i$=1 | | $i$=2 | | $i$=3 | |
|---|---|---|---|---|---|---|
| | $R_1$ | $C_1$ | $R_2$ | $C_2$ | $R_3$ | $C_3$ |
| 1 | 0.6 | 2 | 0.8 | 3 | 0.7 | 1 |
| 2 | 0.7 | 4 | 0.8 | 5 | 0.8 | 2 |
| 3 | 0.9 | 5 | 0.9 | 6 | 0.9 | 3 |

The total capital available is 10 (in units of thousand rupees). Here $x_i$ is the number of

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parallel units placed in the $i$-th subsystem. $R_i$ and $C_i$ be the reliability and cost for $i$-th subsystem.

3. (a) Write down the dual of the primal problem with equality constraints. [2]

(b) Use geometric programming technique, to solve the following constrained optimization problem: [8]

Minimize $z = 0.188x_1x_3$

subject to the constraints

$1.75x_1x_2^{-1}x_3^{-1} \le 1$

and $900x_1^{-2} + x_1^{-2}x_2^2 \le 1$

4. (a) State performance index for minimum energy control system. [2]

(b) State Pontryagin's maximum principle. [2]

(c) Find the stationary path $x = x(t)$ for the functional $J = \int_0^1 (1+\ddot{x}^2)\,dt$ [6]

subject to the boundary conditions

$x(0)=0,\ \dot{x}(0)=1,\ x(1)=1,\ \dot{x}(1)=1.$

5. Write short notes on the following: [5+5]

(i) Laplace crossover, (ii) Ranking selection

6. (a) Discuss the genetic algorithm based constraints handling technique for nonlinear constrained optimization problem. [3]

(b) Write down the advantages of genetic algorithm. [3]

(c) How is the velocity updated in PSO algorithm? Discuss its modification in PSO-Co. [2+2]

**M.A./M.Sc. Semester IV Examination, 2021 (CBCS)**

**Subject: Mathematics (Applied/Pure Stream)**

**Course: MMATAME407-2 (Advanced Operations Research-II)**

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **four** questions. Only **first four** answers will be evaluated. $4\times 10 = 40$

1. Solve the following problem using dynamic programming [10]

$$Maximize\ z = 20x_1 + 50\,x_2 + 60\,x_3$$

subject to $x_1 + 2x_2 + 2x_3 \le 10$, $x_1, x_2, x_3 \ge 1$, where $x_1, x_2, x_3$ are integers.

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2. A man is engaged in buying and selling identical items. He operates from a warehouse that can hold 500 items. He can sell any quantity up to the stock at the beginning of each month. He can purchase as many items as he wants each month, provided that he keeps his inventory below 500 items at the end of each month. He has the following error-free cost sale price forecasts for the next four months: [10]

| Month ($i$) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Cost ($c_i$) | 27 | 24 | 26 | 28 |
| Sale price ($p_i$) | 28 | 25 | 25 | 27 |

Currently, he has 200 units in stock. How many units should he sell and buy over the next four months? Find the solution using dynamic programming.

3. Define percentiles product life of a system. If the unreliability for a component of random variable ***t*** is given as: [2×5]

$$F(t) = \begin{cases} 0 & \text{if } t < 0 \\ 0.5\, t^2 + 0.5\, t & \text{if } 0 \le t \le 1 \\ 1 & \text{if } t \ge 1 \end{cases}$$

Find the hazard rate of the component. What is the median life of this component? Find $B_{10}$ and $B_{70}$ for the life of the components, where $B_\alpha$ life is the time by which $\alpha$ percent of the products fail.

4. (a) Find the mean time between failure (MTBF) for a system of three components, where the components are in parallel and their failure rates are $\alpha_i, i = 1,2,3$, assuming that the time to failure random variables are independent. [5]

(b) Consider a system with seven components, which will work if five of the seven components function ($5 - \text{out} - \text{of} - 7$). Each component's reliability for a given period is 0.92. Find the reliability of the system. [5]

5. Write down the assumptions of the power supply model $(M|M|c):(c|FCFS)$ and then obtain the steady-state solution. [2+8]

6. The following mortality rates have been observed for a certain type light bulbs of 1000 units: [10]

| Week: | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Percentage failing by the end of week: | 5 | 15 | 35 | 57 | 100 |

The replacement of a single bulb costs Rs. 10. The cost of replacing all bulbs at the same time in a group would be Rs. 4. It is proposed to replace all bulbs at fixed intervals, regardless of whether the bulbs have burned out. It is also necessary to replace burned-out bulbs immediately. Determine the time interval at which all bulbs should be replaced.

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**M.A./M.Sc. Semester IV Examination, 2021 (CBCS)**
**Subject: Mathematics (Applied Stream)**
**Course: MMATAME407-6 (Quantum Mechanics-II)**

Time: 2 Hours Full Marks: 40

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Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

Answer any **four** questions. Only **first four** answers will be evaluated. $4\times 10 = 40$

1. Define scattering amplitude. Show that the differential cross section for the non-relativistic scattering of a spinless particle by a short range, central potential is given by modulus square of the scattering amplitude. [2+8]
2. Starting from the radial Schrodinger equation, obtain the Kato identity. [10]
3. Let a one-dimensional harmonic oscillator be in its ground state at $t=-\infty$, which is subjected to a perturbation, $H'(t)=E\,X\,e^{-t^2}$, over an infinite period of time, where $E$ is a constant. Calculate the probability that the oscillator will be in the *n*-th excited state of the unperturbed oscillator. [8+1+1]
4. Show that in the classical limit, solution of the Schrodinger equation corresponding to the motion of a particle of mass $m$ under the potential $V(r)$ describes a fluid (statistical mixture) of non-interacting classical particles of mass $m$ subject to the potential $V(r)$. Determine the density and current density of that fluid. [8+2]
5. Determine the s-wave phase shift for the scattering of a particle of mass $m$ by the attractive square-well potential defined by, [10]
$$\begin{aligned} V(r) &= -V_0 \quad (V_0>0) \text{ for } 0\le r\le a \\ &= 0 \qquad\qquad\quad \text{for } r>a. \end{aligned}$$
6. Derive the Breit-Winger formula for determining resonance. [10]

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**M.A./M.Sc. Semester IV Examination, 2021 (CBCS)**

**Subject: Mathematics (Applied/Pure Stream)**

**Course: MMATPME407-3 (Euclidean and non-Euclidean Geometries-II)**

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **four** questions. Only **first four** answers will be evaluated. $4\times 10 = 40$

1. Write comparisons among plane Euclidean, hyperbolic and elliptic geometries. [10]

2. Prove that the Poincare metric on $H^2$ is invariant under the action of the group $SL(2,\mathbb{R})$. [10]

3. (a) Determine the direction of the tangent vector at any point of $S^2$. [5]

   (b) Prove that any isometry of $S^2$ is induced by the unique linear isometry of $\mathbb{R}^3$. [5]

4. (a) State Hilbert's hyperbolic axiom of parallels. Define real hyperbolic plane. What is the nature of Hilbert plane satisfying Dedekind's axiom? [5]

   (b) Describe Beltrami-Klein model of hyperbolic geometry. [5]

5. (a) Let $L=\{(0,y)\in H^2\}$ and $P=(2,2)$. Show that thereare infinitely many lines through $P$ in $H^2$which do not meet the line $L$. [5]

   (b) Give an example to show that Euclidean distance between two points is finite but hyperbolic distance is infinite. [5]

6. Define a Lorentzian space. Give an example of it. Define concircular and projective transformations on a Riemannian space. What is the geometrical significance of concircular curvature tensor? [$2\times 5$]

**M.A./M.Sc. Semester IV Examination, 2021 (CBCS)**

**Subject: Mathematics (Pure Stream)**

**Course: MMATPME407-5 (Advanced Differential Geometry - II)**

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **four** questions. Only **first four** answers will be evaluated. $4\times 10 = 40$

1. (a) Define almost complex manifold. Show that every almost complex manifold is of even dimension. [2+5]

   (b) Prove that, in an almost complex manifold $M$ of dimension $2m$, the almost complex structure $F$ has $m$ eigenvalues $i$ and $m$ eigenvalues $-i$. [3]

2. Show that a four dimensional conformally flat K$\ddot{a}$hler manifold is locally symmetric. [10]

3. (a) Define a nearly K$\ddot{a}$hler manifold with an example. If the Nijenhuis tensor of a [1+2+3]

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nearly Kähler manifold $M$ vanishes, then show that $M$ is a Kähler manifold.

(b) Let $D$ be an affine connection on an almost complex manifold $M$ with an almost complex structure $F$. When is $D$ said to be $F$-connection? Show that $D$ is $F$-connection if and only if $D_X\bar{Y} = \overline{D_X Y}$ for all $X, Y \in \chi(M)$. [1+3]

4. (a) Let $M(\phi,\xi,\eta,g)$ be a contact metric manifold. Show that $M$ is $K$-contact if and only if $\phi X = -\nabla_X \xi$ for all $X \in \chi(M)$. [6]

(b) Let $M$ be a $K$-contact manifold with contact metric structure $(\phi,\xi,\eta,g)$. Show that the sectional curvature of any plane section of $TM$ containing $\xi$ is equal to -1. [4]

5. Define a Sasakian manifold. Prove that every three-dimensional $K$-contact manifold is Sasakian. [2+8]

6. What do you mean by totally geodesic submanifold and invariant submanifold of a Kenmotsu manifold? Show that an invariant submanifold $\bar{M}$of a Kenmotsu manifold $M^{2n+1}(\phi,\xi,\eta,g)$ is totally geodesic if and only if $\overline{M}$ is parallel. [1+1+8]

**M.A./M.Sc. Semester IV Examination, 2021 (CBCS)**

**Subject: Mathematics (Pure Stream)**

**Course: MMATPME407-6 (Operator Theory and Applications II)**

Time: 2 Hours Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **four** questions. Only **first four** answers will be evaluated. $4\times 10 = 40$

1 (a) Let $E_1$ and $E_2$ be two orthogonal projections on the closed subspaces $M_1$ and $M_2$ of a complex Hilbert space $X$ respectively. Prove that $E_1E_2$ is an orthogonal projection if and only if $E_1E_2 = E_2E_1$. Find also the range of $E_1E_2$ if it is an orthogonal projection. [5]

(b) Let $E_1$ and $E_2$ be two orthogonal projections on the closed subspaces $M_1$ and $M_2$ of a complex Hilbert space $X$ respectively. Prove that $E_1 + E_2$ is an orthogonal projection on $\overline{[M_1 \cup M_2]} = M_1 + M_2$ if and only if $E_1$ is orthogonal to $E_2$. [5]

2 Let $X$ be a complex Hilbert space and let $\{A_n\}$ be a sequence of self-adjoint, commuting transformations from $B(X,X)$. Let $B \in B(X,X)$ be a self-adjoint transformation such that $A_jB = BA_j$ for all $j$ and further suppose that $A_1 \le A_2 \le \cdots. \le A_n \le \cdots. \le B$. Prove that there exists a self-adjoint bounded linear transformation $A$ such that $A_n \to A$ strongly and $A \le B$. [10]

3 State and prove the spectral representation of compact normal operators on Hilbert spaces. [10]

4 (a) Prove that the spectrum of a bounded self-adjoint linear operator $T: H \to H$ on a complex Hilbert space $H$ lies on real axis (i.e., $\sigma(T) \subset R$) [4]

(b) Let $T: H \to H$ be a bounded self-adjoint linear operator on a complex Hilbert space $H(\neq \{\theta\})$. Then prove that $m$ and $M$ are spectral values of $T$ where $m = \underset{\|x\|=1}{inf}\langle Tx, x\rangle$ [6]

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and M= $\sup_{\|x\|=1}\langle Tx, x\rangle$.

5 Let $T: H \to H$ be a bounded self-adjoint linear operator on a complex Hilbert space $H$ ($\neq \{\theta\}$). Also let $E_\lambda$ ($\lambda$ real) be the projection of $H$ onto the null space $N(T_\lambda^+)$ of the positive part $T_\lambda^+$ of $T_\lambda = T - \lambda I$. Then prove that $\xi = (E_\lambda)_{\lambda \in R}$ is a spectral family on the interval $[m, M]$ , where $m = \inf_{\|x\|=1}\langle Tx, x\rangle$ and M= $\sup_{\|x\|=1}\langle Tx, x\rangle$. [10]

6 (a) Prove that the multiplication operator $T: L^2(-\infty, \infty) \supset D(T) \to L^2(-\infty, \infty)$ defined by $x \mapsto tx$ is a self-adjoint linear operator where $D(T) = \{x \in L^2(-\infty, \infty): \int_{-\infty}^{\infty} t^2\, |x(t)|^2 dt < \infty\}$. [5]

(b) Define Caley transformation of a self-adjoint operator $T: D(T) \to H$, where $D(T)$ is a dense subspace of a complex Hilbert space $H$and prove that the Caley transformation is a unitary operator. [1+4]