Internal Assessment
M.A./ M.Sc. Semester-IV Examination,2021(CDOE)
Subject: Mathematics (Pure Stream)(CBCS)

Answer of MMATP401 & MMATP402 together should be limited to two A4 size pages,
Answer of MMATP403 should be limited to two A4 size pages,
Answer of MMATP404 & MMATG405 together should be limited to two A4 size pages,
Answer of MMATPME406-1 should be limited to two A4 size pages,
Answer of MMATP407-6 should be limited to two A4 size pages.

Notation and symbols have their usual meaning.

Time: 2 Hours **Full Marks: 50**

Paper :MMATP 401
(Abstract Algebra-III)

Answer any one question. Only first answer will be evaluated. **1×5 = 5**

1. Let $f(x)$ be a non-constant polynomial over a field K. Prove that there is a splitting field of $f(x)$ over K.
2. Let F be a field and α, β be two roots of an irreducible polynomial over F. If $\alpha \in F(\beta)$, then show that $F(\alpha) = F(\beta)$.

Paper :MMATP 402
(Calculus of $\mathbb{R}^n$-II)

Answer any one question. Only first answer will be evaluated. **1×5 = 5**

1. Let $I = [0,1], Q = I \times I$ and $f: Q \to \mathbb{R}$ be a function, defined by

$$f(x,y) = \frac{1}{q}, \text{ if } x \text{ and } y \text{ are both rationals and } = \frac{p}{q}, \text{ where } p, q \text{ are positive integers with } gcd(p,q) = 1.$$
$$= 0, \text{ otherwise}$$

Show that $\int_Q f$ exists.

2. Let S be a bounded subset of $\mathbb{R}^n$ and $f, g: S \to \mathbb{R}$ be two bounded functions. Suppose f, g are integrable over S. If f and g agree on S except for a set of measure zero, then show that

$$\int_S f = \int_S g.$$

Paper:MMATP403
(Topology-III)

Answer any one question. Only first answer will be evaluated. **1×10 = 10**

1. Prove that a topological space is Hausdorff if and only if each net in the space converge to at most one point.
2. (a) Prove that every quotient space of a discrete space is discrete.
   (b) Prove that a uniform space (X, μ) is T_1 if and only if the intersection of all members of μ is the diagonal in $X \times X$. (5+5)

Paper :MMATP404
(Set theory and Mathematical Logic)

Answer any one question. Only first answer will be evaluated. **1×5=5**

1. **If α, β, γ are cardinal numbers such that $\alpha \leq \beta$, then show that $\alpha + \gamma \leq \beta + \gamma$.**
2. **Prove that the pairs $\{\sim, \wedge\}, \{\sim, \vee\}, \{\sim, \rightarrow\}$ are adequate sets of connectives.**

Paper :MMATG405
(Graph Theory)

Answer any one question. Only first answer will be evaluated. **1×5 = 5**

1. **Prove that a connected graph is a tree if and only if any two vertices x *and* y are connected by a unique path.**
2. **Let $G(V, E)$ be a simple connected graph with n vertices such that $d(v) \geq \frac{n}{2}$ for all $v \in V$. Show that G is Hamiltonian.**

Paper :MMATPME406-1
(Advanced Functional Analysis-II)

Answer any one question. Only first answer will be evaluated. **1×10= 10**

1. **(a) Prove that the set of all non-invertible elements in X is a closed set .**
   (b) Prove that a subspace of a normed linear space is weakly closed if and only if it is strongly closed. **(5+5)**
2. **(a) Show that multiplication operation in a Banach Algebra X is continuous.**
   (b) If f_1 *and* f_2 are multiplicative functionals with the same null space, then show that $f_1 = f_2$. **(5+5)**

Paper :MMATPME407-6
(Operator Theory and Applications-II)

Answer any one question. Only first answer will be evaluated. **1×10= 10**

1. **(a) Show that the residual spectrum of a bounded self-adjoint operator defined on a complex Hilbert space is empty.**
   (b) Let E_1 *and* E_2 be two orthogonal projections on the closed subspaces M_1 *and* M_2 of a complex Hilbert space X respectively. Prove that E_1E_2 is an orthogonal projection if and only if $E_1E_2 = E_2E_1$. Find also the range of E_1E_2 if it is an orthogonal projection . **(5+5)**
2. **Let $T: H \rightarrow H$ be a bounded self-adjoint linear operator on a complex Hilbert space H. Show that**
   (a) all the eigen values of T (if they exist) are real.
   (b) Eigen vectors corresponding to (numerically) different eigen values of T are orthogonal. **(5+5)**

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Answer of MMATAME406-1 should be limited to two A4 size pages,
Answer of MMATAME407-2 should be limited to two A4 size pages.

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Time: 2 Hours **Full Marks: 50**

Paper: MMATA401
(Fluid Mechanics)

Answer any one question. Only first answer will be evaluated. **1×10=10**

1. **State and prove Kelvin's circulation theorem for the motion of an incompressible inviscid fluid in a simply connected region.** **3+7**
2. **State and prove Kutta-Joukowski theorem.** **3+7**

Paper: MMATA402
(Wavelet Analysis)

Answer any one question. Only first answer will be evaluated. **1×5=5**

1. **(a) Give an example of a function $f(x)$ such that $f(x) \in L^1(\mathbb{R})$ but $(x) \notin L^2(\mathbb{R})$. Justify your answer.**
   (b) Define "Support of a function". What do you mean by a "Compact Support"?
   3+1+1
2. **(a) What do you mean by the auto correlation function? What is normalised auto correlation function?**
   (b) Establish the relation between the auto correlation function of a signal $f(t) \in L^2(\mathbb{R})$ and $f(w)$.
   1+1+3

Paper: MMATA403
(Dynamical Systems)

Answer any one question. Only first answer will be evaluated. **1×5=5**

1. **Define conservative dynamical system. Give an example of it. Prove that the phase volume of a conservative system is constant.** **2+1+2**
2. **Write short note on the following:**
   (a) Hopf bifurcation
   (b) Saddle-Node bifurcation **2.5+2.5**

Paper: MMATA404
(Introduction to Quantum Mechanics)

Answer any one question. Only first answer will be evaluated. 1×5=5

1. Show that for the scattering of electromagnetic radiation from a stationary electron, the change in wavelength depends only on the scattering angle.
2. Is it possible for a quantum particle to penetrate a region in which its Newtonian kinetic energy is negative? Support your answer by elaborate calculation with an example.

Paper: MMATG405
(Chaos and Fractals)

Answer any one question. Only first answer will be evaluated. 1×5=5

1. Write down two important properties of fractal objects. Briefly discuss the construction of VonKoch curve and then show that the length of the curve is infinite. 2+3
2. Give Mathematical definition of Chaotic map $f:\mathbb{R}\to\mathbb{R}$. Consider a map g: Unit circle $S \to S$ defined by $g(\theta)=\theta+\alpha$, where the rotation α is irrational. Show that the map g is topologically transitive on $\mathbb{R}$. 2+3

Paper: MMATAME406-1
(Boundary Layer Flows and Magneto-hydrodynamics-II)

Answer any one question. Only first answer will be evaluated. 1×10= 10

1. Deduce the magnetic induction equation in MHD flows. Explain each term of this equation physically. 5+5
2. Write short note on the following:
   (a) Lundquist Criterion
   (b) Alfv'en waves 5+5

Paper: MMATAME407-2
(Advanced Operations Research-II)

Answer any one question. Only first answer will be evaluated. 1×10= 10

1. (a) Derive steady state equations for non-Poisson queuing model ($M/E_k/1 : \infty/FCFS/\infty$).
   (b) What do you mean by group replacement policy? Derive the condition(s) that the group replacement policy becomes preferable than individual replacement policy after a certain time period *t*. 5+(1+4)
2. Write short notes on the following:
   (a) Pontryagin's Maximum Principle
   (b) Dynamic programming 4+6