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M.A./M.Sc. Semester I Examination, 2020 (CBCS, CDOE)
Subject: Mathematics
Course: MMATG103 & MMATG104

Time: 2 Hours Full Marks: 40

Use separate answer booklets for each course. The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[Notation and symbols have their usual meaning]

MMATG103 (Topology I)
[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2 = 20

1. (a) Prove that a co-finite topological space (X, τ) is discrete if and only if X is a finite set. [3]
   (b) Give an example with proper justifications of a continuous bijective mapping which is not a homeomorphism. [2]
   (c) Let (X, τ) be a topological space, $A \subset X$ and $x_0 \in X$. If there exists a sequence $\{x_n\}_{n\in\mathbb{N}}$ in A converging to x_0, then show that $x_0 \in \overline{A}$. Does the converse hold? Support your answer. [2+3]
2. (a) Let (X, τ) and (Y, τ_1) be two topological spaces. If a mapping $f: (X, \tau) \to (Y, \tau_1)$ is continuous, then show that $f(\overline{A}) \subset \overline{f(A)}$ holds for every subset A of X. [3]
   (b) Give an example with proper justification of a T_2 space which is not T_3. [4]
   (c) Prove that every T_4 space is a $T_{3\frac{1}{2}}$ space. [3]
3. (a) Prove that a topological space (X, τ) is normal if and only if for each pair of disjoint closed sets A, B in X, there exists an open set U in X such that $A \subset U$ and $B \cap \overline{U} = \emptyset$. [4]
   (b) Is regularity a hereditary property? Justify your answer. [3]
   (c) Prove or disprove: In every topological space (X, τ), the set A' of all limit points of $A(\subset X)$ is closed in (X, τ). [3]

MMATG104 (Differential Geometry I)
[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2 = 20

1. Define reparametrization of a parametrized curve with an example. Show that any reparametrization of a regular curve is regular. [2+2+6]
2. (a) Calculate the first fundamental form of the surface patch $\sigma(u,v) = a + up + vq$ with p and q being perpendicular unit vectors and a is a constant. [3]
   (b) Let $f : S_1 \to S_2$ be a diffeomorphism, where S_1 and S_2 are two surfaces. Show that f is an isometry if and only if, for any surface patch σ of S_1, the patches σ and $f \circ \sigma$ of S_1 and S_2 respectively, have the same first fundamental form. [7]

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3. (a) Let γ be a unit speed curve in $\mathbb{R}^3$ with constant curvature and zero torsion. Show that γ is a circle or part of a circle. [6]

(b) Compute the torsion of the circular helix $\gamma(\theta) = (a\cos\theta, a\sin\theta, b\theta)$, $\theta \in \mathbb{R}$, where a and b are constants. [4]