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M.A./M.Sc. Semester I Examination, 2020 (CBCS, CDOE)

Subject: Mathematics

Course: MMATG105 & MMATG106

Time: 2 Hours Full Marks: 40

Use separate answer booklets for each course. The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

MMATG105 (Functional Analysis I)

[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. $10\times 2 = 20$

1. (a) When is a metric space said to be first category? Give an example of it with justifications. Let (X,d) be a complete metric space. If X contains no isolated points, then show that X is uncountable. [1+2+3]

   (b) Let (X,d) be a complete metric space and let $T:(X,d)\to(X,d)$ be continuous. If for some positive integer k, $T^k:(X,d)\to(X,d)$ is a contraction map then prove that T has a unique fixed point in X. [4]

2. (a) Suppose $\{x_n\}$ be a sequence in X such that $x_n \to x_0$ as $n\to\infty$ and suppose $\{\alpha_n\}$ be a sequence of scalars such that $\alpha_n \to \alpha_0$ as $n\to\infty$. Show that $\alpha_n x_n \to \alpha_0 x_0$ as $n\to\infty$. [2]

   (b) Define equivalent norms on a linear space. Prove that in a finite dimensional normed linear space any two norms are equivalent. Give an example in support of the statement. [2+4+2]

3. Define a complete orthonormal set of vectors in a Hilbert space. Let X be a Hilbert space and $\{e_k\}$ be an orthonormal set of vectors in X. Suppose that $x\in X$. Prove that the following statements are equivalent. [10]

   i. $\{e_i\}$ is complete.

   ii. $\langle x, e_i\rangle = 0 \Rightarrow x=\theta_X$, θ_X being the zero vector of X.

   iii. $x=\sum_{i=1}^{\infty}\langle x, e_i\rangle\, e_i$

   iv. $\sum_{i=1}^{\infty}|\langle x, e_i\rangle|^2 = \|x\|^2$.

MMATG106 (Linear Algebra)

[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. $10\times 2 = 2$

1. (a) Let W be a subspace of a finite dimensional vector space V over a field F. Show that $\dim(V/W) = \dim V - \dim W$. [6]

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(b) Show that the quadratic form $x^2 + 2y^2 + 3z^2 - 2xy + 4yz$ is indefinite. [4]

2. (a) Let V and W be finite dimensional vector spaces of same dimension and $T \in L(V,W)$. Show that the following statements are equivalent: [6]

    (i) T is one-one

    (ii) T is onto

    (iii) Rank of T = dim of V.

   (b) Find all the possible Jordan canonical forms of a matrix whose characteristic polynomial is given by $(x+5)^2(x-2)$. [4]

3. (a) Show that any two similar matrices have the same minimal polynomial. [6]

   (b) Let $\beta = \{(2,1),(3,1)\}$ be an ordered basis for $\mathbb{R}^2$. Determine the dual basis of β. [4]