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M.A./M.Sc. Semester I Examination, 2020 (CBCS, CDOE)

Subject: Mathematics

Course: MMATG107 (Classical Mechanics-I)

Time: 2 Hours | Full Marks: 40

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

Answer any **four** questions. Only **first four** answers will be evaluated. $10\times 4 = 40$

1. (a) What do you mean by the degree of freedom of system? Show that the constraints in a rigid body are conservative. [2+3]

   (b) Obtain the equation of motion of a simple pendulum by using Lagrangian method and hence deduce the formula for its time period for small amplitude oscillations. [5]

2. (a) Derive Hamilton's canonical equations of motion for a conservative holonomic system. [6]

   (b) Using Hamilton's equations of motion, show that the Hamiltonian $H=\frac{p^2}{2m}e^{-rt}+\frac{1}{2}m\omega^2x^2e^{rt}$ leads to the equation of motion of a damped harmonic oscillator. [4]

3. (a) What do you mean by ignorable coordinate? Discuss the Routh's procedure for ignorable coordinates. [6]

   (b) For a conservative holonomic system, prove that $H=T+V$. [4]

4. (a) Show that the shortest distance between two points on a plane is a straight line. [5]

   (b) Show that for a spherical surface, the geodesics are the great circles. [5]

5. (a) State and prove the principle of least action for a conservative system. [6]

   (b) Find the extremum of the functional $I=\int_{x_1}^{x_2}\left[y'^2+z'^2+2yz\right]dx$ with $y(0)=0; y\left(\frac{\pi}{2}\right)=-1$ and $z(0)=0; z\left(\frac{\pi}{2}\right)=1$. [4]

6. (a) Derive Euler's equation of motion of the rigid body moving with one point of it as fixed. [6]

   (b) If a rectangular parallelopiped with its edges $2a$, $2a$, $2b$ rotates about its centre of gravity under no forces. Prove that, its angular velocity about one principal axis is constant and about the other axis it is periodic, the period about the first axis is $\frac{\left(a^2+b^2\right)}{\left(b^2-a^2\right)}$. [4]