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M.A./M.Sc. Semester I Examination, 2020 (CBCS, CDOE)
Subject: Mathematics
Course: MMATG108 & MMATG109

Time: 2 Hours Full Marks: 40

Use separate answer booklets for each course. The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notation and symbols have their usual meaning]

MMATG108 (Ordinary Differential Equations)
[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2 = 20

1. (a) Show that the function $f(t,x)=\left(x+x^2\right)\dfrac{\cos(t)}{t^2}$ satisfies Lipschitz condition in $|x|\le 1$ and $|t-1|<\dfrac{1}{2}$. [4]

   (b) Find the general solution of the system $\dot{x}=x-5y, \dot{y}=x-3y$. [6]

2. (a) Show that $P_n(-z)=(-1)^n P_n(z)$, where $P_n(z)$ stands for Legendre polynomial of degree n. [4]

   (b) Solve the equation $3z\dfrac{d^2w}{dz^2}+\dfrac{dw}{dz}-2w=0$ in a series about $z=0$. [6]

3. (a) Find the nature of the critical point for the system $\dot{x}=-2x+3y, \dot{y}=-x+y$. [3]

   (b) Convert $\dot{x}=px+qy, \dot{y}=-qx+py$ into its polar form and hence solve it, where p and q are real numbers. Draw phase diagrams for different signs of p and q. [7]

MMATG109 (Partial Differential Equations)
[Marks: 20]

Answer any **two** questions. Only **first two** answers will be evaluated. 10×2 = 20

1. (a) Find the surface satisfying the partial differential equation $\frac{\partial^2 z}{\partial y^2}=6x^3y$ and containing the two lines $y=0=z$ and $y=1=z$. [4]

   (b) Solve the partial differential equation $z^2(p^2+q^2+1)=1$, where $p=\frac{\partial z}{\partial x}$ and q= $\frac{\partial z}{\partial y}$. [6]

2. (a) Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial z^2}$ into parabolic, hyperbolic or elliptic kind. [4]

   (b) Reduce the given partial differential equation into its canonical form and hence solve it. [6]

$$\frac{\partial^2 u}{\partial x^2}-a^2\frac{\partial^2 u}{\partial y^2}=0$$

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3. Solve the partial differential equation $\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \Theta^2} = 0$, [10]

subject to the conditions, $\frac{\partial \varphi}{\partial r} = 0$ at $r = a$ and $\frac{1}{r}\frac{\partial \varphi}{\partial \Theta} = -U sin\Theta$ as $r \to \infty$.