Internal Assessment
M.A./ M.Sc. Semester-I Special Examination,2020(CDOE)
Subject: Mathematics (Old CBCS)

Notation and symbols have their usual meaning.

Time: 2 Hours **Full Marks: 55**

Paper :MMATG101
(Real Analysis-I)

Answer any **one** question**.** Only **first** answer will be evaluated**.** **1×5 = 5**

1. If $\{f_n\}$ is a sequence of measurable function on a measurable set E and if $\lim_{n\to\infty} f_n(x) = f(x)$ for almost all $x \in E$, then show that f is measurable on E .

2. If $f: [a, b] \to \mathbb{R}$ is monotone , then prove that the set of points of discontinuity of f in (a, b) is at most countable.

Paper :MMATG102
(Complex Analysis-I)

Answer any **one** question**.** Only **first** answer will be evaluated**.** **1×5 = 5**

1. Using Liouville's theorem give a proof of the fundamental theorem of algebra.

2. State and prove Morera's theorem.

Paper:MMATG103
(Topology-I)

Answer any **one** question**.** Only **first** answer will be evaluated**.** **1×5= 5**

1. Define T_1-axiom in a Topological space X. Prove that X is T_1 if and only if each singleton set is a closed set in X. (1+4)

2. Show that a continuous function from a compact space to a T_2-space is a homeomorphism provided it is a bijection.

Paper :MMATG104
(Differential Geometry-I)

Answer any **one** question**.** Only **first** answer will be evaluated**.** **1×5=5**

1. Verify the Frenet-Serret formulae of the space curve $\gamma(t) = (\cos t, \sin t, t)$.

2. Obtain a necessary and sufficient condition for a space curve to be a plane curve.

Paper :MMATG105
(Functional Analysis-I)

Answer any **one** question. Only **first** answer will be evaluated. **1×5 = 5**

1. State and prove Baire's category theorem.

2. Define an inner product space. Show that every inner product space X is a normed linear space under a norm to be defined by you . (1+4)

Paper :MMATG106
(Linear Algebra)

Answer any **one** question. Only **first** answer will be evaluated. **1×5=5**

1. Show that every non-null subspace of a finite dimensional inner product space V possesses an orthonormal basis.

2. Show that a real symmetric matrix is positive definite if and only if all of its principal minors are positive.

Paper :MMATG107
(Classical Mechanics-I)

Answer any **one** question. Only **first** answer will be evaluated. **1×10= 10**

1. a) Define generalised coordinates of a dynamical system. Deduce Lagrange's equations of motion of a dynamical system of n degrees of freedom specified by n generalised coordinates $q_k(k = 1,2,3, \dots \dots, n)$ in a conservative field of force. (1+6)

   b) Find the Lagrangian for the motion of a particle of unit mass moving in a central force field under inverse square law of force and obtain an equation connecting the radial coordinate and time t . 3

2. a) State Hamilton's principle for conservative forces and deduce it from D'Alembert's principle. (1+4)

   b) Under what conditions, Hamiltonian of a Dynamical system can be obtained without calculating the Lagrangian? Assuming that such conditions are satisfied, find the Hamiltonian of spherical pendulum and obtain Hamilton's equations of motion. Also find an equation satisfied by one of the coordinates. (1+3+1)

Paper: MMATG108
(Ordinary Differential Equations)

Answer any **one** question. Only **first** answer will be evaluated. **1×5=5**

1. Express $(x^3 - 2x^2 + 3x + 2)$ in terms of Legendre's polynomials.

2. Find the general solution and draw the phase potrait for the linear system

$$\dot{x} = x,$$
$$\dot{y} = -x + 2y$$

   What role do the eigen vectors of the matrix A play in determining the phase portrait? (4+1)

Paper: MMATG109
(Partial Differential Equations)

Answer any **one** question. Only **first** answer will be evaluated. **1×5=5**

1. Describe in brief the method of solving Cauchy's problem of a first order quasilinear partial differential equation in two independent variables.

2. Using Charpit's method, find the complete integral of the partial differential equation $xpq + yq^2 = 1$, where p, q have their usual meanings.

Paper: MMATG110
(C Programming)

Answer any **one** question. Only **first** answer will be evaluated. **1×5=5**

1. a) Define C- variables. Discuss with examples different types of variables used in C. (1+4)

   b) What is an array? 1

2. Write a C-program using a function subprogram to sort an array of integers in ascending order.