Internal Assessment
M.A./ M.Sc. Semester-IV Examination,2020(CDOE)
Subject: Mathematics (Pure Stream)(Old Pattern)

Notations and symbols have their usual meanings

Time: 2 Hours Full Marks: 20

Paper :MPG 401
(Modern Algebra-III)

Answer any one question. Only first answer will be evaluated. **1×5 = 5**

1. Let S be any non-empty set and R be a ring with identity. Prove that there exists a free R-module F such that F has a basis S' equipotent with S.

2. State and Prove the Fundamental theorem of Galois Theory.

Paper :MPG 402
Unit- I
(General Topology-II)

Answer any one question. Only first answer will be evaluated. **1×3 = 3**

1. When is a topological space said to be totally disconnected? Prove that the connected subsets in a totally disconnected space are the singletons. (1+2)

2. Define uniform continuity between two uniform spaces. Prove that a uniformly continuous function is continuous. (1+2)

Unit- II
(Functional Analysis-III)

Answer any one question. Only first answer will be evaluated. **1×2 = 2**

1. When is a sequence $\{x_n\}$ in a normed linear space X said to be weakly convergent to an element x of X? If $x_n \to x$ weakly, then prove that $\{\|x_n\|\}$ is bounded. (1+1)

2. a) When is a linear operation said to be bounded?
   b) Prove that $l_p (1 < p < \infty)$ is reflexive. (1+1)

Paper :MPS 403
(Advanced Functional Analysis-II)

Answer any one question. Only first answer will be evaluated. **1×5=5**

1. **When is an element in a Banach Algebra X with identity called invertible? Prove that the set of all invertible elements in X forms an open set in X.**

   (1+4)

2. **Define weak topology and $weak^*$ topology in a conjugate space of a normed linear space. Prove that the notion of weak convergence of a sequence coincides with the convergence notion of the same sequence arising out of weak topology on a normed linear space.** **(1+1+3)**

Paper : MPS 403
(Differential Geometry of Manifolds-II)

Answer any one question. Only first answer will be evaluated. **1×5=5**

1. **Define an almost complex manifold. Give an example of it.**

2. **Define the Euclidean Connection on $\mathbb{R}^n$. Show that the Geodesics on $\mathbb{R}^n$ with respect to the Euclidean Connection are the straight lines with constant speed parametrizations.**

   (2+3)

Paper :MPS 404
(Operator Theory and Applications-II)

Answer any one question. Only first answer will be evaluated. **1×5=5**

1. **Let X be a normed linear space and $T: X \to X$ be compact linear. Prove that the set of eigen values of T is countable and zero is the only possible point of accumulation of the set of eigen values.**

2. **Let $T \in B(X, X)$, where X is a Banach space. Show that the spectrum $\sigma(T)$ of T is compact.**

Internal Assessment
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Paper: MAG 401
(Continuum Mechanics-III)

Answer any one question. Only the first answer will be evaluated. **1×5=5**

1. **State and prove Kelvin's circulation theorem for the motion of an inviscid fluid in a simply connected region.**

2. **Using Cisotti's equation, find the complex potential for simple harmonic progressive gravity waves(water waves).**

Paper : MAG 402
Unit-I
(Elements of Quantum Mechanics)

Answer any one question .Only the first answer will be evaluated. **1×3=3**

1. **State the Broglie's hypothesis. Hence explain the concept of wave particle duality.** **(1+2)**
2. **What do you mean by position probability density of a quantum particle? Using Schrodinger equation, prove that the total probability is conserved.** **(1+2)**

Unit-II
(Chaos and Fractals)

Answer any one question .Only the first answer will be evaluated. **1×2=2**

1. **Define non-hyperbolic fixed point of a map with example.**

2. **Write a short note on 'Topological transivity'.**

Paper : MAS 403
(Viscous Flows, Boundary Layer Theory & Magneto-Hydrodynamics-II)

Answer any one question. Only the first answer will be evaluated. **1×5=5**

1. **Discuss briefly 'sausage mode of instability'.**

2. **Interpret physically the expression for Lorentz force per unit volume(simplified form). Hence indicate the existence of transverse Alfven's waves.**

Paper : MAS 404

(Advanced Operations Research-II)

Answer any one question. Only the first answer will be evaluated. **1×5=5**

1. **(a) Write down the dual of the primal problems with equality constraints.**

   (b) Write down the computational procedure for solving an optimization problem by Dynamic programming technique.

   (2+3)

2. **Write short notes on the following:**

   (a) Classification on queuing models.

   (b) Bellman's principle of optimality.

   (2+3)