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M.A./M.Sc. Semester IV Examination, 2020 (Old Pattern under CDOE)

Subject: Mathematics (Applied Stream)

Paper: MAG 401 (Continuum Mechanics III)

Time: 2 Hours — Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[*Notation and symbols have their usual meanings*]

Answer any **five** questions. Only the **first five** answers will be evaluated. 5×9 = 45

1. (a) Write down the stress tensor at a point for an inviscid fluid. [2]

   (b) Obtain Euler's dynamical equations in vector invariant form for an incompressible inviscid fluid. [7]

2. (a) Show that circulation round a circuit in a moving fluid is equal to the flux of vorticity across any surface bounded entirely by the circuit. [6]

   (b) The velocity components for a fluid motion are given by $u = a(x^2 - y^2), \upsilon = 2a(x^2 - xy), w = 0$. Obtain the circulation about the circle: $x^2 + y^2 = 1, z = 0$ for this motion. [3]

3. (a) State and prove Kelvin's minimum energy theorem. [2+3]

   (b) Applying Milne-Thomson circle theorem, find the image of a source outside a circle. [4]

4. Define vortex surface. State and prove Helmphltz first theorem for vortex motion. [2+2+5]

5. (a) Explain the physical significance of group velocity. [3]

   (b) Show that for waves on deep water, the group velocity is half the wave velocity and on very shallow water, it is equal to the wave velocity. [3+3]

6. Derive Navier-Stokes' equations of motion of a viscous incompressible fluid in vector form. Define Reynolds number and interpret it physically. [5+2+2]

7. (a) An incompressible viscous fluid flows along an elliptic pipe under uniform axial pressure gradient. Find the rate of mass flux through it. [4]

   (b) For an incompressible viscous fluid, obtain the expression for the rate of energy dissipation due to viscosity. [5]

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M.A./M.Sc. Semester IV Examination, 2020 (Old Pattern under CDOE)

Subject: Mathematics (Pure Stream)

Paper: MPG 401 (Modern Algebra III)

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[*Notation and symbols have their usual meanings*]

Answer any **five** questions. Only the **first five** answers will be evaluated. $5\times9 = 45$

1. (a) Let $p(x) = x^3 + 9x + 6$ is irreducible in $\mathbb{Q}[x]$. Let θ *be* a root of $p(x)$. Find the inverse of $1 + \theta$ in $\mathbb{Q}(\theta)$. [3+2]

   (b) Suppose α is a rational root of a monic polynomial in $\mathbb{Z}[x]$. Prove that α is an integer. [4]

2. (a) Let F be a field and α be a algebraic over F . Show that $\{f \in F[x]: f(\alpha) = 0\}$ is a principal ideal of $F[x]$. [4]

   (b) Show that algebraic closure of $\mathbb{Q}$ is an infinite extension of $\mathbb{Q}$. [5]

3. Deduce the splitting field of $x^3 - 2$ over $\mathbb{Q}$. Find the degree of its extension. Is it a Galois extension? Justify your answer. [4+2+3]

4. (a) Let K be finite extension of F . Prove that K is a splitting field over F if and only if every irreducible polynomial in $F[x]$ that has a root in K splits completely in $K[x]$. [3+3]

   (b) Show that every irreducible polynomial over a field of characteristic 0, is separable. [3]

5. (a) Find all automorphisms of $\mathbb{Q}\left(\sqrt[3]{3}, \sqrt{7}\right)$. [5]

   (b) Let F be the splitting field of $x^{2021} - 2$ over $\mathbb{Q}$. Show that there exists a subfield E of F with $[F:E] = 43$. Further show that F is the Galois extension of E. [2+2]

6. (a) Determine the Galois group of $(x^2-2)(x^2-3)(x^2-5)$. Determine all the subfields of the splitting field of this polynomial. [4+2]

   (b) Show that the Galois group of cyclotomic field $\mathbb{Q}(\xi_n)$ of n^{th} roots of unity is isomorphic to the multiplicative group $(\mathbb{Z}/n\mathbb{Z})^\times$ of non-zero residue class of integers modulo n. [3]

7. Let *R* be a ring and let *M* be an *R*-module. Then show that *M* is Noetherian *R*-module if and only if every non-empty set of submodules of *M* contains a maximal element under inclusion if and only if every submodule of *M* is finitely generated. [3+3+3]