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M.A./M.Sc. Semester IV Examination, 2020 (Old Pattern under CDOE)

Subject: Mathematics (Applied Stream)

Paper: MAG 402

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[*Notation and symbols have their usual meanings*]

Write the answer to Questions of each Group in separate books.

Group - A (Elements of Quantum Mechanics)

[Marks: 27]

Answer any **three** questions. Only the **first three** answers will be evaluated. 3×9 = 27

1. State Bohr's postulates on hydrogen-like atoms. Hence calculate the energy levels of the hydrogen atom. [4+5]

2. (a) Show that the energy E must exceed the minimum value of potential V (x), for any normalizable solution of the time-independent Schrodinger equation in one dimension. [5]

   (b) If a wave function is normalized at t = 0, show that it will remain normalized for all t. [4]

3. Calculate the energies and normalised eigenstates for a particle within an infinite potential box. [9]

4. Show that in the n-th eigenstate of the harmonic oscillator, the average of kinetic energy $< T >$ is equal to the average of potential energy $< V >$. [9]

5. From the postulates of Quantum Mechanics, show that the expectation value of an observable A in the state $|\Psi>$ is, $<A> = <\Psi|A|\Psi>$. Hence show that if A is time-independent and commutes with the corresponding Hamiltonian, then $<A>$ is a constant of motion. [5+4]

Group - B (Chaos and Fractals)

[Marks: 18]

Answer any **two** questions. Only **first two** answers will be evaluated. 2×9 = 18

1. (a) Give the mathematical definition of a chaotic map. [4]

   (b) Show that the conjugacy is an equivalence relation. [5]

2. (a) Prove that the logistic map has a 2- cycle when the growth parameter exceed 3. [5]

   (b) Calculate the Lyapunov exponent for the tent map. [4]

3. Write two important properties of fractal objects. What do you mean by self - similar dimension of fractals ? Determine the self - similar dimension of von Koch curve. [2+3+4]

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Group – A (General Topology- II)

[Marks: 27]

Answer any **three** questions. Only **first three** answers will be evaluated. 9×3 = 27

1. (a) Is $\mathbb{R}_\ell$ connected? Support your answer. ($\mathbb{R}_\ell$ denotes Sorgenfrey line i.e. $\mathbb{R}$ with lower limit topology). [3]

   (b) Prove that a topological space (X, τ) is connected if and only if $Bd(A) \neq \emptyset$ for every nonempty proper subset A of X. ($Bd(A)$ denotes boundary of $A \subset X$). [4]

   (c) Let $f: \mathbb{R} \to \mathbb{R}$ be any function such that $f(\mathbb{Q}) \subset \mathbb{R} - \mathbb{Q}$ and $f(\mathbb{R} - \mathbb{Q}) \subset \mathbb{Q}$. Show that f can not be continuous. [2]

2. (a) Let (X, τ) be a topological space. Prove that each point of X is contained in exactly one component of X. [5]

   (b) Define a locally connected space. Prove that every compact locally connected space has a finite number of components. [1+3]

3. (a) Let (X^*, σ) be an one point compactification of a topological space (X, τ), where $X^* = X \cup \{\infty\}$. Prove that (X, τ) is a compact subspace of (X^*, σ) if and only if ∞ is an isolated point of (X^*, σ). [4]

   (b) Prove that every compact metric space is sequentially compact. [5]

4. (a) Define a uniformity μ on a non-empty set X. [2]

   (b) Let μ be a uniformity on a nonempty set X and $\tau_\mu = \{G \subset X: \text{for each } x \in G,$ there exists $U \in \mu$ such that $U(x) \subset G\}$. Show that τ_μ is a topology on X. [4]

   (c) Give an example with proper justification of topological space which is not metrizable. [3]

5. (a) Let $\mathbb{R}^n$ be the Euclidean n-space and $x_0 \in \mathbb{R}^n$. Show that $\pi_1(\mathbb{R}^n, x_0) = \{0\}$, where 0 is the identity element of the group $\pi_1(\mathbb{R}^n, x_0)$. [4]

   (b) Show that the map $p: \mathbb{R} \to S^1$ given by the equation $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map, where S^1 is the unit circle in $\mathbb{R}^2$ endowed with the topology induced by the usual topology of the plane $\mathbb{R}^2$. [5]

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Group - B (Functional Analysis - III)

[Marks: 18]

Answer any **two** questions. Only **first two** answers will be evaluated. 9×2 = 18

1. (a) Let x_0 be a non-zero vector in the normed linear space X over the field F $(= \mathbb{R}\ or\ \mathbb{C})$. Prove that there exists a bounded linear functional g, defined on X, such that $\|g\| = 1$ and $g(x_0) = \|x_0\|$. [3]

   (b) For every x in a normed linear space X over $F(= \mathbb{R}\ or\ \mathbb{C})$, show that $\|x\| = \sup\{\frac{|f(x)|}{\|f\|} : f \in X^/,\ f \neq 0\}$. (Here $X^/$ denotes the dual space of X). [4]

   (c) Is l^1 reflexive? Support your answer. [2]

2. (a) Prove that every finite dimensional normed linear space is reflexive. [3]

   (b) Give an example with proper justification of a sequence in a normed linear space which is weakly convergent but not strongly convergent. [3]

   (c) Let X be an inner product space over $\mathbb{C}$ and A be an orthonormal set of vectors in X. Prove that for each $x \in X$, the set $E_x = \{ z \in A : < x, z > \neq 0 \}$ is a countable set. [3]

3. (a) Let S be a closed subspace of a Hilbert space X. Prove that every $x \in X$ can be expressed uniquely as $x = y + z,\ y \in S,\ z \in S^{\perp}$, where $S^{\perp}$ stands for orthogonal complement of S . [3]

   (b) State and prove Riesz representation theorem over a Hilbert space. [1+5]