M.A./M.Sc. Semester IV Examination, 2020 (Old Pattern under CDOE)
Subject: Mathematics (Applied Stream)

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[*Notation and symbols have their usual meanings*]

Paper: MAS 403 (Viscous Flows, Boundary Layer Theory & Magneto Hydrodynamics-II)

Answer any **five** questions. Only the **first five** answers will be evaluated. $5\times9 = 45$

1. Write short notes on [3×3]
(a) magnetic pressure number (b) Sausage mode of instability (c) MHD Couette flow.

2. State and prove Cowling theorem in MHD. [2+7]

3. Show that magnetic force-free field satisfies Helmholtz equation and obtain its general solution. [3+6]

4. (a) Using MHD approximations, simplify the expression for total current. [4]
(b) Prove that for a perfectly conducting fluid, lines of magnetic force are frozen in the fluid. [5]

5. Derive magnetic induction equation and explain each term of the equation physically. [5+4]

6. Set up the governing equations with appropriate boundary conditions for MHD Rayleigh problem. Mention the assumptions involved in it. [3×3]

7. (a) Show that the magnetic Reynolds number is the ratio of induced magnetic field to the applied magnetic field, in case of Hartmann flow. [4]
(b) Obtain Ohm's law with Hall current for a static conductor. [5]

M.A./M.Sc. Semester IV Examination, 2020 (Old Pattern under CDOE)
Subject: Mathematics (Pure Stream)

Time: 2 Hours Full Marks: 45

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[*Notation and symbols have their usual meanings*]

Paper: MPS 403 (Advanced Functional Analysis-II)

Answer any **five** questions. Only the **first five** answers will be evaluated. 5×9 = 45

1. Let X be a normed linear space and Y be a subspace of X. Define a best approximation to an element $x \in X$ out of Y. Let X be an inner product space and let $x \in X$. Prove that $y \in Y$ is a best approximation to x if and only if $(x-y) \perp Y$. Moreover show that $d(x,Y) = \sqrt{\langle x, x-y \rangle}$. [1+6+2]

2. (a) Let X be a complex Banach algebra with an identity e. If $x \in X$ and $p(t)$ is a polynomial with complex coefficients then show that $\sigma(p(x)) = p(\sigma(x))$, where $\sigma(x)$ is the spectrum of x. [9]

3. (a) Let X be a complex commutative Banach algebra with an identity e. When is an ideal M of X said to be maximal? Show that any proper ideal of X is contained in a maximal ideal of X. [1+3]

   (b) Define an invertible element in a Banach algebra X with identity e. Prove that the set of all invertible elements of X is an open set in X. [1+4]

4. Let X be a complex commutative Banach algebra with an identity e .and let M denote the set of all maximal ideals in X. Define the Gelfand topology on M. Prove that the maximal ideal space M is a compact Hausdorff space with respect to its Gelfand topology. [2+7]

5. (a) Let X be a complex commutative Banach algebra with an identity e. For $x \in X$ define the spectral radius of x. If $r(x)$ is the spectral radius of x then prove that $r(x) = \lim_{n\to\infty} \|x^n\|^{\frac{1}{n}}$. [1+8]

6. (a) Let X be a complex Banach algebra with an identity e. When is a non-empty subset I of X said to be an ideal of X? Show that if I is a proper ideal of X, then so is its closure, $\bar{I}$. [1+3]

   (b) Let I be a proper closed ideal of a complex Banach algebra X with an identity e. Show that the quotient space X/I is also a commutative Banach algebra with identity $e+I$ satisfying $\|e+I\| = 1$. [5]

7. Prove the representation theorem for a bounded linear functional over $C[a,b]$, the space of all real valued continuous functions over $[a,b]$ with sup norm. [9]

Paper: MPS 403 (Differential Geometry of Manifolds - II)

Answer any **five** questions. Only the **first five** answers will be evaluated. $5\times9 = 45$

1. (a) Define Riemannian manifold. Deduce the Koszul's formula on a Riemannian manifold. [2+5]

   (b) State Hopf-Rinow theorem for a connected Riemannian manifold. [3]

2. (a) Show that every 3-dimensional Einstein manifold is a manifold of constant curvature. [6]

   (b) Prove that a Riemannian manifold of dimension greater than 3 is of constant curvature if and only if it is projectively flat. [4]

3. (a) What do you mean by a metric connection on a Riemannian manifold? [2+5]

   Let ∇ be a metric connection on a Riemannian manifold (M, g) and $\overline{\nabla}$ be another linear connection given by $\overline{\nabla}_X Y = \nabla_X Y + T(X,Y)$ for all $X, Y \in \chi(M)$, where T is the torsion tensor corresponding to ∇. Show that the following conditions are equivalent:

   (i) $\overline{\nabla} g = 0$

   (ii) $g(T(X,Y),Z) + g(Y,T(X,Z)) = 0.$

   (b) If f is a smooth function on a Riemannian manifold (M, g) with Riemannian connection ∇ such that $f = g(X, X)$ for all $X \in \chi(M)$ then show that grad f = 2 $\nabla_X X$. [3]

4. (a) Deduce Gauss and Codazzi equations for the submanifold of a Riemannian manifold. [6]

   (b) Define hypersurface and lines of curvature. [2+2]

5. (a) Deduce Ricci identity on a Riemannian manifold. [6]

   (b) Define Ricci tensor on a Riemannian manifold. Show that it is symmetric. [2+2]

6. (a) Define an almost complex structure F on a real smooth manifold. Find the rank of F . [2+3]

   (b) Show that an almost complex structure in an almost complex manifold is not unique. Does the sphere S^3 admit a complex structure? Support your answer. [3+2]

7. (a) If a vector field V in an almost complex manifold is strictly almost analytic then show that $N(V, X) = 0$ for every X , where N is the Nijenhuis tensor. [5]

   (b) Show that there always exists a half symmetric $F-$connection on an almost complex manifold. [5]