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M.A./M.Sc. Semester IV Examination, 2020 (Old Pattern under CDOE)
Subject: Mathematics (Applied Stream)

Time: 2 Hours Full Marks: 45

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
[*Notation and symbols have their usual meanings*]

Paper: MAS 404 (Advanced Operations Research-II)

Answer any **five** questions. Only the **first five** answers will be evaluated. 5×9 = 45

1. Solve the following problem using dynamic programming [9]

$$\text{Minimize } z = x_1^2 + x_2^2 + x_3^2 + x_4^2$$

subject to $x_1 + x_2 + x_3 + x_4 = d(> 0)$, $x_1, x_2, x_3, x_4 \geq 0$.

2. Using dynamic programming technique to solve the linear programming problem [9]
Minimize $z = x_1 + x_2$ subject to $2x_1 + x_2 \geq 4, x_1 + 7x_2 \geq 7, x_1, x_2 \geq 0$.

3. A truck can carry a total of 10 tons of products. There are three types of products available for shipment. Their weights and values are tabulated in the following table. If at least one of each type of product must be shipped, then use dynamic programming to determine the quantities that will maximize the total value. [9]

Types	Values (Rs.)	Weights (tons)
A	20	1
B	50	2
C	60	2

4. (a) Show that if the inter arrival times are exponentially distributed, then the number of arrivals in a time period is a Poisson process. [3]
(b) Explain the following topics: [2×3]
(i) queue discipline, (ii) arrival process, (iii) service process

5. (a) Find the (i) average number of customers in the system, and (ii) average queue length for the queue model $(M|M|1):(N|FCFS)$. [2+2]
(b) In a railway marshaling yard, goods trains arrive at a rate of 30 trains per day. Assume that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes. If the yard has a capacity of 9 trains, calculate [2+3]
(i) the probability that the yard is empty.
(ii) average queue length.

6. Write down the assumptions of the machine servicing model$(M|M|R):(K|K|GD), K > R$ and then obtain the steady-state solution. [2+7]

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7. Using Geometric programming, solve the following problem [9]

Minimize $z_x = 2x_1x_2^{-3} + 4x_1^{-1}x_2^{-2} + \frac{32}{3}x_1x_2$ subject to $10\, x_1^{-1}x_2^{2} = 1$.

M.A./M.Sc. Semester IV Examination, 2020 (Old Pattern under CDOE)
Subject: Mathematics (Pure Stream)

Time: 2 Hours Full Marks: 45

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[*Notation and symbols have their usual meanings*]

Paper: MPS 404 (Operator Theory and Applications II)

Answer any **five** questions. Only the **first five** answers will be evaluated. 5×9 = 45

1. (a) When is a sesquilinear functional defined on a Hilbert space said to be bounded? If f is a bounded sesquilinear functional on a Hilbert space X then prove that [1+4]

$$\|f\| = \sup_{\|x\|=\|y\|=1} |f(x,y)|$$

(b) If f is a bounded symmetric, sesquilinear functional on a Hilbert space then prove that $\|f\| = \|\hat{f}\|$ where $\hat{f}$ denotes the quadratic form associated with f. [4]

2. (a) Define spectrum and approximately spectrum of a bounded linear operator, $T : X \supset D_T \rightarrow Y$; X, Y being two normed linear spaces. Show that $\sigma_p(\mathrm{T}) \cup \sigma_c(\mathrm{T}) \subset \pi(\mathrm{T})$. [2+2]

(b) Let X be a complex Banach space and $T \in B(X, X)$. Show that the spectrum of T is bounded. [5]

3. (a) Show that (i) $R_\mu - R_\lambda = (\mu - \lambda)R_\mu R_\lambda$ and (ii) $R_\lambda R_\mu = R_\mu R_\lambda$ where $\lambda, \mu \in \rho(T)$, the resolvent set of $T \in B(X, X)$, X being a Banach space. [2+2]

(b) Let X be a normed linear space and $T: X \rightarrow X$ be compact linear. If X is infinite dimensional then show that $0 \in \sigma(T)$. [5]

4. Let X be a normed linear space and $T: X \rightarrow X$ be compact linear. Prove that for every $\lambda(\neq 0)$ the range of $T - \lambda I$ is closed. [9]

5. (a) Let X be a complex Hilbert space and $T \in B(X, X)$ be normal. Prove that $r_\sigma(T) = \|T\|$ and there is some $\lambda \in \sigma(\mathrm{T})$ such that $|\lambda| = \|T\|$. [4]

(b) Let X be a complex Hilbert space and $T \in B(X, X)$. Then prove that the following statements are equivalent. [5]

(i) There is some $\lambda \in \pi(\mathrm{T})$ such that $|\lambda| = \|T\|$.

(ii) $\|T\| = \sup_{\|x\|=1} |\langle Tx, x\rangle|$

6. State and prove the spectral theorem of compact normal operators on Hilbert spaces. [9]

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7. (a) Prove that the spectrum of a bounded self-adjoint linear operator $T: H \to H$ on a complex Hilbert space H lies in the closed interval $[m, M]$ where $m = \inf_{\|x\|=1} \langle Tx, x \rangle$ and M= $\sup_{\|x\|=1} \langle Tx, x \rangle$. [4]

(b) Let $T: H \to H$ be a bounded self-adjoint linear operator on a complex Hilbert space $H\ (\neq \{\theta\})$. Then prove that m and M are spectral values of T where $m = \inf_{\|x\|=1} \langle Tx, x \rangle$ and M= $\sup_{\|x\|=1} \langle Tx, x \rangle$. [5]