Internal Assessment
M.A./ M.Sc. Semester-II Examination,2021(CDOE)
Subject: Mathematics (Old CBCS)

(Answers of each paper should be limited to two A4 size pages)

Notation and symbols have their usual meanings.

Time: 2 Hours **Full Marks: 55**

Paper :MMATG201
(Real Analysis-II)

Answer any **one** question. Only **first** answer will be evaluated. **1×5 = 5**

1. If $\{f_n\}$ is a sequence of measurable function on a measurable set E and if $\lim_{n\to\infty} f_n(x) = f(x)$ for almost all $x \in E$, then show that f is measurable on E.

2. Let f be a Lebesgue integrable periodic function with period 2π. If f is a function of bounded variation in a neighbourhood of x, prove that the Fourier series of f converges at x to $\frac{1}{2}\{f(x+) + f(x-)\}$.

Paper :MMATG202
(Complex Analysis-II)

Answer any **one** question. Only **first** answer will be evaluated. **1×5 = 5**

1. State and prove the Argument principle .

2. Using Cauchy's residue theorem evaluate $\oint_{|z|=2} \frac{dz}{(z-1)^2(z^2+9)}$.

Paper:MMATG203
(Topology-II)

Answer any **one** question. Only **first** answer will be evaluated. **1×5= 5**

1. Prove that the image of a locally connected space under a mapping which is both open and continuous is locally connected . Does this result hold if the function is only continuous ? Support your answer .

2. State and prove Urysohn's metrization theorem .

Paper :MMATG204
(Differential Geometry-II)

Answer any **one** question. Only **first** answer will be evaluated. **1×5=5**

1. Determine the geodesics on a circular cylinder by solving the geodesic equations.

2. State and prove Meusnier's theorem.

Paper :MMATG205
(Calculus of $\mathbb{R}^n$-I)

Answer any **one** question. Only **first** answer will be evaluated. **1×5 = 5**

1. State and prove Cantor's intersection theorem over $\mathbb{R}^n$.

2. Let $f: \mathbb{R}^3 \to \mathbb{R}^2$ be a function defined by
$$f(x, y, z) = (xyz, 2x + 3y + 4z), \forall (x, y, z) \in \mathbb{R}^3 .$$
Find $Df(1,2,0)$.

Paper :MMATG206
(Abstract Algebra-I)

Answer any **one** question. Only **first** answer will be evaluated. **1×5=5**

1. Let G be a finite group of order p^n , where p is prime and $n > and \neq 0$. Then show that G has non-trivial center $Z(G)$.

2. Let G be a non-commutative group of order $2p$, p odd prime . Show that the order of the group of inner automorphisms of G is $2p$.

Paper :MMATG207
(Operations Research)

Answer any **one** question. Only **first** answer will be evaluated. **1×10= 10**

1. a) Use revised simplex method to solve the following LPP:
Maximize $z = 5x_1 + 4x_2$
Subject to $6x_1 + 4x_2 \leq 24 , x_1 + 2x_2 \leq 6 , -x_1 + x_2 \leq 1 , x_2 \leq 2 , x_1, x_2 \geq 0$

   b) Discuss the effect of discrete change of requirement vector on the optimal solution of an LPP.
(5+5)

2. Consider the following LPP:
Max $z = cx$
Subject to $Ax \leq b , x \geq 0 , where \;\; c, x^T \in \mathbb{R}^n , b^T \in \mathbb{R}^m \; and$
$$A \;\; is \;\; m \times n \;\; real \; valued \; coefficient \; matrix .$$
Determine the range of the discrete changes of the components a_{kj} of the coefficient matrix A , which does not belong to basis , so as to maintain the optimal feasible solution of the LPP .

Paper: MMATG208
(Integral Transforms)

Answer any **one** question. Only **first** answer will be evaluated. **1×5=5**

1. Determine the Fourier transform of $e^{-\frac{x^2}{2}}$.

2. Is $f(t) = e^{t^2}$ a function of exponential order ? Justify your answer .

Paper: MMATG209
(Integral Equations)

Answer any **one** question. Only **first** answer will be evaluated. **1×5=5**

1. Convert the boundary value problem $\frac{d^2y}{dx^2} + xy = 1\,, y(0) = 0\,, y(1) = 1$ into an integral equation .

2. Solve the integral equation $u(x) = x + \int_0^x (t - x)u(t)dt$.

Paper: MMATG210(Theory)
(Numerical Methods)

Answer any **one** question. Only **first** answer will be evaluated. **1×5=5**

1. Discuss the power method to find the largest eigen value of a square matrix of order n .

2. Discuss Milne's method for solving initial value problem .